Market Segmentation With Nonlinear Pricing

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Abstract

This paper studies the effect of captive consumers in a competitive model of nonlinear pricing. We focus on the benefits and drawbacks of allowing what we call market segmentation, namely, a situation where the price-quality menu offered to captive consumers can differ from that offered to consumers that are exposed to competition. We find that the effect of market segmentation depends on the relationship between the range of consumer preferences found in captive markets and that found in competitive markets. When the range of consumer preferences in captive markets is “wide”, segmentation is quality and (aggregate) welfare reducing, while the opposite holds when the range of consumer preferences in captive markets is “narrow”. Segmentation always harms captive consumers, while it always benefits consumers located in competitive markets.

JEL Classifications: D43, L1.

1 Introduction

The use of second-degree (or indirect) price discrimination by firms competing in oligopolistic settings is a phenomenon that has been increasingly recognized and documented.¹ This paper contributes to the recent body of theoretical literature on this subject, by studying competition within a horizontally differentiated duopoly.² Differently from previous studies, we assume that

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²The growing number of empirical papers documenting the existence of second-degree price discrimination in competitive environments include Shepard (1991), Borenstein (1991) and Busse and Rysman (2005).

consumers vary in their degree of captivity to a given firm, as well as in their unobservable valuation for quality. The presence of captive consumers plays an important role in many industries, such as for instance the telecommunication industry or the market for industrial gas (Armstrong and Vickers 1993). The purpose of this paper is that of identifying how the presence of captive consumers conditions the nature of the competitive equilibrium when differentiated firms compete in price-quality menus. This has important policy implications, by informing the debate of whether market segmentation (where firms discriminate between captive and non-captive markets) should or should not be allowed. This is often a controversial issue. Armstrong and Vickers (1993) for instance cite the example of the United Kingdom’s industrial gas market, where, prior to 1988 British Gas (BG) was free to set prices to its customers without constraints. Customers without an alternative source of energy complained that they were being charged more than the less captive consumers. Finally, a Monopolies and Mergers Commission Report removed BG’s freedom to discriminate between more and less captive consumers.

A detailed description of the model is provided in section 2, while section 3 establishes some useful preliminaries. An important feature of our framework is that a consumer’s marginal valuation for quality is inversely related to the distance between the consumer’s ideal product-space location and that of the firm from which he purchases. This assumption has also been made elsewhere in the literature, e.g. Spulber (1989), Stole (1995) and Hamilton and Thisse (1997). To see what this assumption entails, consider for instance a theatre, selling tickets for opera performances. Here, the distance between the consumer and the firm captures the consumer’s appreciation of opera as a genre – with a lower distance representing greater appreciation. Take two consumers: a serious opera lover and one who is less of a connoisseur. Our assumption implies that, for lousy opera companies, the difference in the maximum price that the connoisseur and the non-connoisseur are willing to pay to see the performance is relatively small. For high quality opera companies, however, the difference between the two consumers is much more apparent. The price that a serious opera lover is willing to pay to see, say, Placido Domingo is much higher than that of someone who has a more tepid attitude towards opera. An implication of this is that consumers who purchase higher quality goods also have stronger brand preferences. This is consistent with the empirical observation that, in several markets, consumers who purchase higher qualities are more brand-loyal than those who purchase lower qualities, and are therefore less likely to switch supplier to take

3Although we concentrate on quality provision, the model could equivalently be used to model situations where quality is homogenous, and the relevant contractible variable is quantity rather than quality. See also Rochet and Stole (2002, footnote 5) on this point.
advantage of small price differences.\textsuperscript{4}

We assume that each consumer belongs to one of three categories: some only have access to one firm, some only have access to the other, and some have access to both – i.e., they belong to the “competitive market”. When facing a given firm, consumers therefore vary along two dimensions. First, they vary with respect to their marginal valuations for quality of the product sold by the firm. Second, they vary in their captivity status – they are either entirely captive to the firm, or they have costless access to both firms. Captive consumers may be located in specific geographical markets (as e.g. in Rosenthal 1980), or they may have higher search costs than other consumers (as e.g. in Salop and Stiglitz 1977). Alternatively, they may be repeat-purchase consumers who are now locked-in (as e.g. in Klemperer 1987).

We allow the range of consumer valuations found in the competitive and the captive markets to differ. While in the competitive market the lowest-valuation consumer for each firm has valuation $v_1$, in the firm’s captive market the lowest-valuation consumer has valuation $v - s$, for some $v > 1$ and $s \leq 1$.\textsuperscript{5} The parameter $s$ describes the range of consumer preferences found in captive markets relative to the range of preferences found in competitive markets.

We compare two cases. In the first case (presented in section 4), market segmentation is allowed, namely firms are allowed to discriminate between consumers located in their captive market and those located in the competitive market. In the second case (presented in section 5), market segmentation is not allowed, so consumers located in captive markets and those located in the competitive market must be offered the same price-quality menu of contracts.

We identify two effects of disallowing segmentation: a \textit{competition effect} and a \textit{discontinuity effect}. The discontinuity effect arises because, when segmentation is not allowed, consumers from different markets (captive/competitive) are lumped together. Although within each market consumers are distributed uniformly (and therefore continuously), when consumers from the two markets are lumped together the resulting density function exhibits a discontinuity. Keeping everything else equal, this lowers the quality offered to consumers with sufficiently low valuations.

The competition effect arises because competition in the competitive market has the by-product effect of softening the efficiency/informational rents trade-off firms face when selecting the quality of low-valuation captive consumers. Keeping everything else equal, this raises the quality offered

\textsuperscript{4}This is for instance well documented within the car market, as shown in Goldberg (1995), Berry, Levinsohn and Pakes (1995), Feenstra and Levinsohn (1995). Indeed, Verboven (1996) calls this feature a “stylized fact of this market”.

\textsuperscript{5}We assume that, in both competitive and captive markets, the highest-valuation consumer for each firm has valuation $v$. 


to these consumers.

Which of these two effects dominates depends crucially on the value of $s$. In particular, the results obtained when captive markets are “narrow” ($s < 1/2$) are radically different from those obtained when they are “wide” ($s > 1/2$). For wide captive markets the competition effect always dominates. Hence, forbidding segmentation raises the the qualities offered to a whole set of consumers. These results are in contrast with the findings of Spulber (1989), Stole (1995) and Hamilton and Thisse (1997), where competition simply shifts rents, but does not affect quality allocations. We show that, with captive markets, this may no longer be true.

If captive markets are sufficiently narrow, though, the competition effect disappears altogether, so disallowing segmentation only brings about the discontinuity effect. Hence, in this case forbidding segmentation lowers the qualities offered to a whole set of types – namely, consumers with sufficiently low valuations.

Section 6 compares the quality allocations obtained under the segmentation and the no-segmentation régimes. In section 7, we characterize their welfare properties. Again, these change in narrow and wide captive markets. For narrow captive markets, aggregate welfare is higher when segmentation is allowed, while the opposite occurs for wide captive markets. Moreover, segmentation always harms captive consumers, while it benefits those located in the competitive market. Section 8 concludes. All the proofs that are not in the main text can be found in the appendix.

2 The model

There are two firms, denoted as firm $L$ and firm $R$, positioned at the left and right extremities of a Hotelling line of length 1, and a continuum of consumers of total mass one. A consumer’s preferences are determined by his ideal product-space location on the line (his location, in short), which is his private information. Let $z$ be the (absolute value of the) distance between a consumer’s ideal product-space location and the firm’s location. Similarly to Spulber (1989), Stole (1995) and Hamilton and Thisse (1997), we assume that the utility of a consumer who purchases quality $q$ at price $p$ is equal to:

$$u(z, p, q) = (v - z) q - p.$$ (1)

for some $v \in \mathbb{R}^+$. In what follows, we refer to $(v - z)$ as the consumer’s valuation. If a consumer does not consume the good at all, his utility is equal to 0. In order to simplify the analysis, we impose the following restriction on $v$:

**Assumption A1** $v \geq 3.$
This assumption is sufficient to ensure that at equilibrium all consumers are served by one of the two firms. As made clear by (1), our setting differs from textbook models of horizontal differentiation. Take for instance a firm selling a product of quality \( q \) at price \( p \). Consider the “standard” textbook approach (for simplicity, let the marginal “transportation cost” be unity). The utility of consuming the good for a consumer located at distance \( z \) from the firm is \( vq - p - z \). Hence, the difference in utility between a consumer located at distance \( z_0 > 0 \) and a consumer located at distance \( z_1 > z_0 \) from the firm is \( z_1 - z_0 \), independent of quality \( q \). By contrast, in our model this difference is \( q(z_1 - z_0) \), increasing in \( q \). A good of zero quality is equally worthless for both consumers. As quality increases, the utility obtained by the consumer at distance \( z_0 \) grows faster than that obtained by the consumer located at distance \( z_1 \).

In addition to having different valuations, consumers also differ in their degrees of captivity to the firms. In order to keep the analysis simple, we consider an extreme case: some consumers are entirely captive to firm \( L \) (they belong to firm \( L \)'s captive market), some consumers are entirely captive to firm \( R \) (they belong to \( R \)'s captive market), and some consumers have access to both firms (they belong to the competitive market). Firms can observe if a consumer is captive or if he is exposed to competition. Whether they can utilize this information when making offers to consumers, though, depends on whether segmentation is or is not allowed. If it is not, firms must allow all consumers to select their favorite choice from the same menu of price-quality offers. By contrast, if segmentation is allowed then firms can offer different deals to captive consumers compared to consumers who are exposed to competition.\(^6\) We assume that consumers are equally split between the three markets. This implies that the mass of consumers in each market (competitive market, \( L \)'s captive market and \( R \)'s captive market) is equal to \( \frac{1}{3} \). We also make the following assumption.

**Assumption A2** In the competitive market, the consumers’ ideal product-space locations are uniformly distributed on \([0, 1]\). In \( L \)'s captive market, they are uniformly distributed on \([0, s]\), while in \( R \)'s captive market they are uniformly distributed on \([1 - s, 1]\), for some \( s \in (0, 1) \).

Assumption A2 makes clear that the range of consumer valuations found in the competitive and the captive markets may differ. Consider for instance firm \( L \) (the case for firm \( R \) is analogous). The ideal product-space locations of its captive consumers belong to \([0, s]\). By contrast, those of

\(^6\)This is presumably what British Gas did before captive consumers successfully filed a complaint with the Monopolies and Mergers Commission, as reported by Armstrong and Vickers (1993).
consumers exposed to competition belong to \([0, 1]\). Note that, at equilibrium, firm \(L\) does not serve all the consumers who are exposed to competition, but only those whose ideal product-space locations belong to \([0, 1/2]\). When \(s < (>) 1/2\), this is larger (narrower) than the range of product-space locations of the consumers located in \(L\)’s captive market.

We assume that the two firms are perfectly symmetric, with quadratic costs of production. The profit from selling quality \(q\) at price \(p\) is then \(p - q^2/2\).

Conditional on a consumer with valuation \(v - z\) purchasing quality \(q\) at price \(p\), joint surplus from trade is equal to \(S(q, z) = (v - z)q - q^2/2\). The full information first-best allocation that maximizes \(S(q, z)\) is given by \(q^{FB}(z) = v - z\). This is the benchmark against which we evaluate the efficiency of quality allocations that result under asymmetric information.\(^7\)

The timing of the game is as follows. At \(t = 1\), the firms simultaneously make price-quality offers; at \(t = 2\), consumers located in the competitive market choose which firm to consume from (if any) and which quality to purchase, while consumers located in the captive markets decide whether or not to purchase from the unique firm they have access to, and (if yes) which quality to purchase; at \(t = 3\) market transactions take place; finally, at \(t = 4\) payoffs are realized.

Throughout the analysis we concentrate on symmetric, pure strategy, deterministic equilibria. We also concentrate on equilibria where firms only offer those qualities that they actually sell.\(^8\) In what follows we therefore use the term equilibrium with these conditions left implicit.\(^9\)

### 3 Preliminaries

Before we turn to the analysis, it is instructive to establish some preliminary results. Let \(z_i\) indicate the distance between a consumer’s ideal product-space location and that of firm \(i = L, R\). The precise value of \(z_i\) is a consumer’s private information. From the revelation principle, our search for the optimal contract can be restricted to direct revelation mechanisms that are incentive compatible. The following lemma characterizes the conditions that must be satisfied in order to have incentive compatibility. For any given mechanism offered by firm \(i\), we indicate the indirect

\(^7\)Note that since the value of \(s\) determines the distribution of consumer valuations in the captive markets, it also affects the maximum surplus achievable. However, our analysis studies the effects of different regimes (discriminatory/non discriminatory) keeping \(s\) fixed, so this is not an issue here.

\(^8\)This restriction is introduced to avoid “out of equilibrium” offers that may complicate things, without really adding to the results.

\(^9\)D’Aspremont, Gabzewicz and Thisse (1979) show that, in the linear-cost specification of the Hotelling model, a pure-strategy price equilibrium may fail to exist when firms are located sufficiently close to the centre of the segment. This occurs because demand functions are discontinuous. For some price configurations, all consumers in firm \(i\)’s turf switch to firm \(j\) for a small reduction in \(p_j\). In our framework, this potential problem is ruled out, since firms are located at the extremities of the Hotelling segment.
utility of a consumer located at distance $z_i$ from firm $i$ who truthfully reveals it as $u_i(z_i)$.\footnote{Consumers differ along two dimensions, namely their distance $z_i$ from the firm, and also their market location (captive/competitive). However, conditional on purchase from firm $i = L, R$, the preferences of a consumer located at distance $z_i$ from firm $i$ are independent of the market (captive/competitive) to which the consumer belongs. Accordingly, when segmentation is not allowed our search for optimal contracts may be restricted to environments where all consumers with the same $z_i$ who purchase from the same firm are offered the same contract. [When segmentation is allowed, firms are free to condition their contractual offer on the market (captive/competitive) to which a consumer belongs, so incentive compatibility along this dimension is not an issue.] This point is also discussed in section 5.}

**Lemma 1:** The following conditions are necessary and sufficient for incentive compatibility:

\[(IC.1) \quad u'_i(z_i) = -q_i(z_i)\]

\[(IC.2) \quad q_i(z_i) \text{ is non-increasing in } z_i.\]

**Proof:** The proof of lemma 1 is standard and is therefore omitted. See for instance Fudenberg and Tirole (1991, Chapter 7).

Condition (IC.1) is the first order condition for local incentive compatibility, while condition (IC.2) is the second-order condition. Together, these two conditions ensure global incentive compatibility. From (IC.1), we see that in order to preserve incentive compatibility, firms must offer higher rents to high valuation (i.e., low $z_i$) consumers. These rents are increasing in the quality offered to low valuation consumers. This is the familiar efficiency/informational rents trade-off first identified by Mussa and Rosen (1978). As is standard in the literature – see for instance Bolton and Dewatripont (2005), p.85 – in what follows we will first characterize each firm’s optimization problem ignoring the monotonicity constraint (IC.2), and then reinstate monotonicity whenever the solution of the unconstrained problem fails to satisfy it. Denote firm $i$’s contractual offer when a consumer declares his distance from the firm’s location to be $\hat{z}$ as $(p_i(\hat{z}), q_i(\hat{z}))$. Conditional on a consumer’s truthfully declaring his distance to be $z_i$, firm $i$’s profit when contracting with this consumer is given by $p_i(z_i) - q_i(z_i)^2 / 2$. Substituting for $p_i(z_i) = (v - z_i) q_i(z_i) - u_i(z_i)$ this becomes $(v - z_i) q_i(z_i) - u_i(z_i) - q_i(z_i)^2 / 2$. We now turn to the analysis.

## 4 Market segmentation is allowed

We start off by investigating the nature of the equilibrium in the case where market segmentation is allowed. Each firm can discriminate between consumers located in its captive market, and those located in the competitive market. This is especially easy if the two markets are divided geographically. Even when this is not the case, though, firms may nonetheless discriminate between captive and non-captive consumers by tailoring their offers to consumers’ characteristics. This is a
widespread practice, which has become increasingly common. As argued by Liu and Serfes (2004), “recent advances in information technology and software tools (..) have taken price discrimination to a new level”. These authors cite a series of examples of the so-called practice of dynamic pricing, where consumers pay different prices according to their demographics, purchasing history and so on. Shaffer and Zhang (2000) provide examples of firms targeting promotions at consumers with greater exposure to rivals.

Note that although firms can discriminate between captive and non-captive consumers, within each market a consumer’s valuation is his private information. The situation is therefore one where firms use a combination of direct and indirect price discrimination, offering nonlinear price schedules that are conditional on the market (captive/competitive) where the consumer is located.

**Captive market.** In its captive market, each firm is a monopolist. The distance between the consumer with lowest valuation and the firm is \( s \), while that between the consumer with the highest valuation and the firm is 0. Since the total mass of consumers located in each captive market is \( \frac{1}{3} \), consumer density is equal to \( \frac{1}{3s} \). Firm \( i = L, R \)’s programme is to maximize

\[
\int_0^s \frac{1}{3s} \left( (v - z_i) q_i(z_i) - u_i(z_i) - \frac{q_i(z_i)^2}{2} \right) dz_i
\]

subject to incentive compatibility (IC.1) and participation: \( u_i(z_i) \geq 0 \) for all \( z_i \in [0, s] \). The problem faced by the firm is a standard screening problem, à la Mussa and Rosen (1978). It is straightforward to show that the optimal contract satisfies: \( u_i(s) = 0 \) and

\[
q_i(z_i) = v - 2z_i.
\]

**Competitive market.** In the competitive market, the distance between the consumer with lowest valuation and the firm is 1, while that between the consumer with the highest valuation and the firm is 0. Firm \( i = L, R \)’s programme is to maximize

\[
\int_0^1 m_i(z_i) \left( (v - z_i) q_i(z_i) - u_i(z_i) - \frac{q_i(z_i)^2}{2} \right) dz_i
\]

subject to incentive compatibility (IC.1), where \( m_i(z_i) \) indicates the density of consumers with distance \( z_i \) purchasing from firm \( i \). Let \( B_i(z_i) \) denote the utility which a consumer located at distance \( z_i \) from firm \( i \) can obtain by purchasing from firm \( -i = \{ L, R \} \setminus i \). This is the consumer’s reservation utility when contracting with firm \( i \). A consumer will purchase from firm \( i \) only if \( u_i(z_i) \geq B_i(z_i) \). What is the value of \( B_i(z_i) \)? Since the total length of the Hotelling segment

\[11\]If this was not the case, the firms would be able to operate first-degree price discrimination.
is equal to 1, \( z_i = 1 - z_{-i} \) for \( i = L, R \) and \(-i = \{L, R\} \setminus i\). A consumer with a high valuation when dealing with \( L \) also has a low valuation when dealing with firm \( R \), and vice-versa.\(^{12}\) We can therefore write
\[
\begin{align*}
B_i (z_i) &= \max (0, u_{-i} (z_{-i})) = \max (0, u_{-i} (1 - z_i)).
\end{align*}
\] (5)

Hence, \( B_i' (z_i) \) is either equal to 0 or it is equal to \(-u'_{-i} (1 - z_i)\). From lemma 1, \(-u'_{-i} (1 - z_i) = q_{-i} (1 - z_i) \geq 0\), where \( q_{-i} (1 - z_i) \) denotes the product quality which a consumer located at distance \( z_{-i} = 1 - z_i \) is offered when contracting with firm \(-i\). This implies that \( B_i' (z_i) \geq 0 \), and brings us to the following lemma.

**Lemma 2:** Given firm \(-i\)’s contract, a consumer located in the competitive market buys from firm \( i \) if and only if \( z_i \leq z_i^m \), where \( z_i^m \) satisfies: \( u_i (z_i^m) = B_i (z_i^m) \), and \( u_i (z_i) > (\text{respectively,} <) B_i (z_i) \) for all \( z_i < (\text{respectively,} >) z_i^m \).

**Proof:** Consider firm \( i \)’s problem. To attract a consumer with distance \( z_i \) located in the competitive market, it must offer at least \( B_i (z_i) \). Since the competitive market is fully covered, we must also have \( q_i (z_i) > 0 \). From lemma 1, this implies that at equilibrium the utility of consumers located in the competitive market purchasing from firm \( i \) is strictly decreasing in \( z_i \). Given \( B_i' (z_i) \geq 0 \), this implies that the consumers’ participation constraint in the competitive market binds at a single point.□

Within the competitive market the farthest consumer served by firm \( i \) is located at distance \( z_i^m \) from the firm. Substituting for \( u_i' (z_i) \) from (IC.1), we can write
\[
\begin{align*}
u_i (z_i) - \int_{z_i}^{z_i^m} q_i (x) dx &= B_i (z_i^m).
\end{align*}
\] (6)

Each firm \( i \)’s problem consists of selecting a marginal type \( z_i^m \) and quality allocations \( q_i (z_i) \) to maximize the firm’s expected payoff subject to incentive compatibility, taking the rival’s contractual offers as given.\(^{13}\) By analogy with Spulber (1989), Stole (1995) and Hamilton and Thisse (1997), it is easy to show that, at equilibrium, the qualities offered to consumers in the competitive market are identical to those offered to their counterparts in the captive markets. Intuitively, for firm \( i \) the only difference between the captive and the competitive market arises because, in the latter, the consumer’s reservation utility may be different from zero. Since \( B_i' (z_i) \) is positive, however, this does not affect the firm’s trade-off between efficiency and informational rents when

\(^{12}\)Note that since the relationship between \( z_L \) and \( z_R \) is one-to-one, the value of \( z_i \), \( i = L, R \) provides a full characterization of a consumer’s preferences for \( i = L, R \) and \(-i = \{L, R\} \setminus i\).

\(^{13}\)In what follows, the expression “firm \( i \)’s marginal type” will be used to indicate the distance \( z_i^m \) of the farthest consumer served by firm \( i \) in the competitive market.
selecting quality. Conditional on type $z^m_i$ purchasing from firm $i$, all consumers located at distance smaller than $z^m_i$ from firm $i$ also necessarily purchase from that firm.\textsuperscript{14} The trade-off involved in offering higher or lower quality is then the same as that faced by a monopoly. Hence, in this environment, competition shifts rents away from firms to consumers, but does not affect quality allocations (and aggregate welfare). As will become clear below, this is in contrast with the case where segmentation is not allowed.

In any symmetric equilibrium, the marginal consumer in the competitive market is located midway between the two firms. Hence, $z^m_L = z^m_R = 1/2$. The full characterization of the equilibrium of the game is found by substituting for $z^m_i = 1/2$ in the firm’s first order condition for $z^m_i$ and then imposing symmetry in the two firms’ contractual offers. This yields $u(1/2) = (v - 1)(v - 2)/2$. The following proposition summarizes our results.

**Proposition 1** (Characterization of equilibrium contract when segmentation is allowed.) When market segmentation is allowed, the equilibrium quality allocations in both the captive and the competitive market are the same, and are given by (3). In the competitive market, $u_i(1/2) = (v - 1)(v - 2)/2$, $i = L, R$.

### 5 Market segmentation is not allowed

We now turn to the case in which market segmentation is not allowed. Consumers located in the competitive and captive markets must be offered the same menu of price-quality contracts. Conditional on purchase from firm $i = L, R$, the preferences of a consumer located at distance $z_i$ from firm $i$ are independent of the market (captive/competitive) to which a consumer belongs. Accordingly, in our search for the optimal contracts, we consider direct revelation mechanisms of the form \{$q_i(z_i), p_i(z_i)$\}.\textsuperscript{15}

Since $s \leq 1$, for each firm the farthest available consumer is located in the competitive market, at distance $1$ from the firm. Firm $i$’s problem is to maximize

$$\int_0^1 m_i (u_i (z_i), z_i) \left( (v - z_i)q_i (z_i) - u_i (z_i) - \frac{q_i (z_i)^2}{2} \right) dz_i$$

subject to incentive compatibility (IC.1) and a participation constraint.

From Lemma 2, we know that consumers located in the competitive market purchase from firm $i$ only if their distance $z_i$ from the firm is $\leq z^m_i$. By contrast, captive consumer have no choice but

\textsuperscript{14}Note that this would not necessarily be true if $B'(z_i)$ was negative.

\textsuperscript{15}This is without loss of generality given that we restrict attention to mechanisms that do not involve randomization – this point is also made by Rochet and Stole (2002), pp.282–283.
purchase from firm $i$. The density of consumers located at distance $z_i$ from firm $i$ who purchase from firm $i$ is therefore equal to:

- if $z_i < s$
  \[
  m_i(z_i) = \begin{cases} 
  (1 + s)/3s & \text{for } z_i \in [0, z_i^m] \\
  1/3s & \text{for } z_i \in (z_i^m, s] \\
  0 & \text{for } z_i > s 
  \end{cases}
  \tag{8}
  \]

- if $z_i > s$
  \[
  m_i(z_i) = \begin{cases} 
  (1 + s)/3s & \text{for } z_i \in [0, s] \\
  1/3 & \text{for } z_i \in (s, z_i^m] \\
  0 & \text{for } z_i > z_i^m 
  \end{cases}
  \tag{9}
  \]

In both (8) and (9) the density $m_i(z_i)$ has a downward discontinuity (at $z_i = z_i^m$ and $z_i = s$, respectively). To see why, consider for instance (8) – the case of (9) is analogous. Here, purchasing consumers with distance $z_i > z_i^m$ from firm $i$ may only be located in the captive market – if they were located in the competitive market, they would not be purchasing from $i$. In what follows, we refer to these consumers as being “unambiguously captive”. By contrast, purchasing consumers with distance $z_i \in [0, z_i^m]$ may be either located in the captive market or in the competitive market.\textsuperscript{16} Clearly, the latter have a greater density than unambiguously captive consumers, and, as a result, we have a downward discontinuity. As will become clear below, this discontinuity plays an important role in our analysis.

In a symmetric equilibrium, $z_i^m = z_i^R = 1/2$. When $s < 1/2$ (i.e., the distribution of preferences in the captive market is “sufficiently narrow”) at equilibrium each firm’s marginal type is also the farthest consumer being served. This is however no longer true when $s > 1/2$ (i.e., the distribution of preferences in the captive market is “sufficiently wide”). The distinction between these two cases turns out to be crucial for characterizing the equilibrium contract. The case where $s < 1/2$ is qualitatively equivalent to a situation in which the captive market is entirely absent. In contrast, the case where $s > 1/2$ possesses novel features, that derive from the presence of a captive market. As a shorthand, in what follows we indicate $s < 1/2$ as the narrow captive markets case, and $s > 1/2$ as the wide captive markets case.

\textsuperscript{16}When purchasing consumers located at a distance $z_i$ from a firm may originate both from the firm’s captive market and from the competitive market, the density $m_i(z_i)$ is given by the sum of $1/3s$ (the density of consumers in the captive market) and $1/3$ (the density of consumers in the competitive market). This yields $(1 + s)/3s$.
5.1 “Wide” captive markets: $s > 1/2$

In this section we characterize the properties of the optimal contract whenever $s > 1/2$. In this case, the farthest consumer served by each firm at equilibrium is located at distance $s$ from the firm. The marginal density of consumer distances being faced by each firm is given by (8). To solve for the equilibrium contracts, we derive each firm's optimal mechanism conditional on $z_m^i < s$.\(^{17}\)

Ignoring monotonicity concerns, each firm $i$ solves (7) subject to (IC.1). Rearranging (6) for any $z_i$ we can write

$$u_i(z_i) = B_i(z_i^m) - \int_{z_i^m}^{z_i} q_i(x)dx. \quad \text{(10)}$$

Substituting for $u_i(z_i)$ into the firm’s objective function, we see that the firm’s problem can equivalently be expressed as one of selecting quality allocations and the marginal type $z_m^i < s$ to solve

$$\max_{q_i(z_i), \; z_i^m} \int_0^s \frac{1}{3} \left( (v - z_i) q_i(z_i) - \frac{q_i(z_i)^2}{2} + \int_{z_i^m}^{z_i} q_i(x)dx \right) \left( \frac{1}{s} + I(z_i) \right) dz_i - B_i(z_i^m) \int_0^s \frac{1}{3} \left( \frac{1}{s} + I(z_i) \right) dz_i \quad \text{(11)}$$

where $I(z_i)$ is given by

$$I(z_i) = \begin{cases} 
1 & \text{for } z_i \leq z_i^m \\
0 & \text{for } z_i > z_i^m
\end{cases} \quad \text{(12)}$$

subject to the captive consumers’ participation constraint\(^{18}\)

$$u_i(z_i) \geq 0 \forall z_i \in [0, s]. \quad \text{(13)}$$

Since the reservation utility of all captive consumers is equal to 0, the necessary and sufficient condition for (13) to hold is that $u_i(s) \geq 0$ or, equivalently, that

$$\int_{z_i^m}^{s} q_i(x)dx - B_i(z_i^m) \leq 0. \quad \text{(14)}$$

After integration by parts, the Lagrangian for the problem can be written as

$$\int_0^s \frac{1}{3} \left( q_i(z_i) \left( v - 2z_i + s \left( 1 - 3\lambda \right) \left( 1 - I(z_i) \right) \right) - \frac{q_i(z_i)^2}{2} \right) \left( \frac{1}{s} + I(z_i) \right) dz_i - \frac{1}{3} B_i(z_i^m) (z_i^m + 1 - 3\lambda) \quad \text{(15)}$$

\(^{17}\)At equilibrium, $z_i^m = 1/2 < s$. Hence, although the conditions we derive below do not guarantee overall optimality, they need to hold true at equilibrium.

\(^{18}\)Participation in the competitive market is determined by the choice of $z_i^m$. 

12
where $\lambda \geq 0$ is the Lagrange multiplier for constraint (14). The unconstrained quality allocations – derived by ignoring condition (IC.2) – are\footnote{As discussed below, these allocations actually violate condition (IC.2).}

- for $z_i \in [0, z_i^m]$
  \[ q_i(z_i) = v - 2z_i \] (16)

- for $z_i \in (z_i^m, s]$
  \[ q_i(z_i) = v - 2z_i + s(1 - 3\lambda) \] (17)

Consumers located at distance $z_i \in (z_i^m, s]$ are unambiguously captive, while those located at distance $z_i \in [0, z_i^m]$ from the firm may be either located in the captive or the competitive market. For these latter consumers the unconstrained quality allocations that emerge when segmentation is not allowed are identical to those that would emerge if segmentation was allowed. For ambiguously captive consumers, the two quality allocations differ by $s(1 - 3\lambda)$. As shown in the appendix, in our framework this value is always positive. Hence, disallowing segmentation induces the firm to offer higher qualities to low-valuation consumers than it would otherwise (i.e., if segmentation was allowed). In order to fully appreciate the different effects at play when selecting the optimal contract, it is instructive to analyze the benchmark case where captive markets are entirely absent, yet the density of $z_i$ is given by (8), as it is here. This is done in the following section.

5.1.1 A benchmark case

The aim of this section is that of separating the direct effect of disallowing segmentation in the presence of captive markets from the indirect effect, which arises because the presence of captive markets changes the distribution of consumer preferences faced by the firms. This indirect effect can be replicated by simply letting the density of $z_i$ be given by (8), which is what we do here. Note that since we are assuming that captive markets are entirely absent, here the marginal type is also the farthest consumer being served by each firm. Hence, in this benchmark the firm’s marginal type is $s$, and $z_i^m$ simply represents the point where the marginal density of $z_i$ has a (exogenously given) discontinuity.

The analysis closely follows that presented in section 4. Each firm selects quality allocations to solve:

\[ \max_{q_i(z_i)} \int_0^s \left( \frac{1}{3} \left( (v - z_i) q_i(z_i) - \frac{q_i(z_i)^2}{2} - u_i(z_i) \right) \left( \frac{1}{s} + I(z_i) \right) \right) dz_i \]
subject to incentive compatibility and participation, where \( I(z_i) \) is given by (12). It is straightforward to verify that the optimal quality allocations prescribe:

- for \( z_i \in [0, z_{i}^{m}] \)
  \[
  q_i(z_i)^{\text{benchmark}} = v - 2z_i
  \]  
  \[(18)\]

- for \( z_i \in (z_{i}^{m}, s] \)
  \[
  q_i(z_i)^{\text{benchmark}} = v - 2z_i - z_{i}^{m}s.
  \]  
  \[(19)\]

The quality allocations described in (18) and (19) exhibit a downward discontinuity at \( z_i = z_{i}^{m} \). This arises from the discontinuous density of consumers faced by each firm. Intuitively, under (8) the standard efficiency-informational rents trade-off bites more severely for low-valuation consumers (i.e., with distance greater than \( z_{i}^{m} \) from the firm) than for higher-valuation ones. When serving low-valuation consumers, firms are therefore more willing to sacrifice quality, in order to save on the informational rents to be offered to higher-valuation consumers.

### 5.1.2 Two effects

In the benchmark case analyzed above, captive markets are entirely absent, but the marginal density of \( z_i \) is exogenously assumed to satisfy (8). This allows us to distinguish two different effects that operate in our framework. The first effect arises because, when segmentation is not allowed, consumers from different markets (captive/competitive) are lumped together, and the resulting density function exhibits a discontinuity. We call this effect the *discontinuity effect* of disallowing segmentation. As seen above, this effect pushes the quality offered to unambiguously captive consumers downwards (leaving the qualities offered to other consumers unchanged).\(^{20}\)

Disallowing segmentation also generates another effect – what we call the *competition effect*. Similar to the discontinuity effect, the competition effect only affects the qualities offered to consumers who are unambiguously captive. However, this effect operates in the opposite direction.\(^{21}\)

The basic idea behind the competition effect is that the consumer benefits generated by competition in the competitive market generate positive spillovers also for unambiguously captive consumers. These positive spillovers are especially relevant, since they operate on the quality levels that are offered to these consumers. As a result, total surplus is also affected. To see how this happens, consider first high-valuation consumers (located at distance smaller than \( z_{i}^{m} \) from the firm).

\(^{20}\) Recall that here unambiguously captive consumers are those located at distance greater than \( z_{i}^{m} \).

\(^{21}\) This can be seen by comparing (16) and (17) with their counterparts derived in section 5.1.1, namely (18) and (19). While for \( z_i \in [0, z_{i}^{m}] \) there is no difference (in both cases quality allocations are identical to those in section 4), for \( z_i \in (z_{i}^{m}, s] \) the quality allocations in (17) exceed those in (19).
these include consumers belonging to the competitive market, competition between the two firms ensures that they obtain a good deal (i.e., yielding high utility) when purchasing the good. Hence, for these consumers, competition brings a direct utility benefit. The quality levels they are offered at equilibrium are however the same as those that would be offered by a monopolist, since (as discussed in section 4) competition does not affect the trade-off faced by firms when selecting quality to offer these consumers – it only affects the price. What about consumers with lower valuations? These consumers are unambiguously captive, so for them competition brings no direct benefit. However, for these consumers competition does nonetheless have an indirect effect. Intuitively, since through competition high-valuation consumers are receiving high rents anyway, the qualities offered to low-valuation consumers can be raised without fear that they may attract higher-valuation types, for whom they are not intended.\textsuperscript{22} In other words, competition for higher valuation consumers softens the efficiency/informational rents trade-off faced by the firm when selecting the qualities offered to lower-valuation consumers. This induces the firm to raise the qualities it offers these consumers. Importantly, the competitive effect arises because the firm is constrained to offer all consumers the same price/quality menu from which to select, and would not arise otherwise.

Overall, therefore, quality allocations are determined by the interplay between two effects – namely the discontinuity effect and the competition effect – moving in opposite directions. The discontinuity effect lowers the qualities offered to low-valuation, unambiguously captive consumers, while the competition effect pushes them upwards. Under assumption A1 – details can be found in the appendix – the competition effect is always sufficiently strong to ensure that the unconstrained quality schedule (given by (16) and (17)) exhibits an upward jump.\textsuperscript{23} However, since it violates condition (IC.2), an upward jump in the quality schedule is incompatible with incentive compatibility. The second-best quality allocation therefore prescribes pooling over some interval $[z_0, z_1]$, where $z_1 > 1/2 > z_0$ (since at equilibrium $z_1^m = 1/2$). The interval $[z_0, z_1]$ is derived by trading off allocative efficiency and rent extraction with respect to the pooling interval as a whole.

Note that at equilibrium the quality allocations offered to $[z_0, s]$ are \textit{above} those that would be offered by a monopolist or if segmentation was allowed. Indeed, some types in $(z_0, s]$ may even be offered quality allocations that are above the first best level. In this case, disallowing segmentation

\textsuperscript{22}Recall that, from (IC.1), in our environment the informational rents that must be offered to preserve incentive compatibility are always decreasing in $z_i$. Firms must prevent high valuation consumers from understating their valuations, not vice-versa.

\textsuperscript{23}The upward jump emerges because the competition effect bites in a discontinuous fashion. It raises the qualities offered to consumers that are located \textit{exclusively} in captive markets while it leaves unchanged the qualities offered to consumers that may be located in \textit{both} the competitive and the captive market.
may generate a new type of distortions, namely upward distortions. The following proposition summarizes our findings. A fuller characterization of the equilibrium conditions can be found in the appendix.

**Proposition 2** (Characterization of equilibrium contract when segmentation is not allowed and \( s > 1/2 \).) When segmentation is allowed and \( s > 1/2 \), pooling always emerges at equilibrium; that is, at equilibrium \( \lambda < 1/3 \), and the same quality \( \bar{q} \) is offered to all types in \([z_0, z_1]\), where \( z_0 < 0.5 < z_1 \) and \( \bar{q} \) satisfies

\[
\int_{z_0}^{z_1} (v - 2z_i + s (1 - 3\lambda) (1 - I(z_i)) - \bar{q} \frac{1}{s} + I(z_i))dz_i = 0. \tag{20}
\]

where \( I(z_i) \) is given by (12). The qualities offered to \( z_i \in (z_1, s] \) and \( z_i \in [0, z_0) \), \( i=L,R \) are given by (17) and (16), respectively. Moreover, \( u_i (1/2) = \bar{q}(v + 6\lambda - 3.5) - \bar{q}^2/2, \) \( i = L, R \).

5.2 “Narrow” captive markets: \( s < 1/2 \)

We now consider the case of narrow captive markets. This is rather different from the case where captive markets are wide, seen above. In particular, at equilibrium the farthest consumer being served by each firm is located in the competitive market (at distance \( z_i^m \) from the firm’s location, where you may recall that at equilibrium \( z_i^m = 1/2 \)). This eliminates the competition effect, since this effect can only bite for those consumers located at distance greater than \( z_i^m \). The trade-off faced by the firm when selecting quality is therefore the same as in section 5.1.1, the only difference between the two cases being that here the marginal density of \( z_i \) is given by (9) instead of (8). To sum up, therefore, when \( s < 1/2 \), the competition effect of disallowing segmentation disappears, while the discontinuity effect remains. However, this effect takes a different form from the previous section, since it alters the qualities offered to consumers in \((s, z_i^m] \), i.e. consumers unambiguously located in the competitive market (while in the previous section it operated on the qualities of consumers unambiguously located in the captive market).

The equilibrium quality allocations satisfy:

- for \( z_i \in [0, s] \)

\[
q_i (z_i) = v - 2z_i \tag{21}
\]

- for \( z_i \in (s, z_i^m] \)

\[
q_i (z_i) = v - 2z_i - 1. \tag{22}
\]
In this case, therefore – and in contrast with the case where \( s > 1/2 \) – the standard monopoly result of underprovision of quality always carries through to the duopoly scenario. Note however that here quality allocation exhibit a downward discontinuity at \( z_i = s \). This is a consequence of the discontinuous density of consumers, described in (9). In contrast with what obtained in section 5.1, here this discontinuity does not violate condition (IC.2).

**Proposition 3** (Characterization of equilibrium contract when segmentation is not allowed and \( s < 1/2 \).) When segmentation is allowed and \( s < 1/2 \), pooling never emerges at equilibrium, and the quality allocations are characterized by (21) and (22). Also, \( u_i (1/2) = \max\{0, (v - 2) (v - 5)/2\} \), \( i = L, R \).

6 Discussion: effect of segmentation on quality allocations

Comparing the quality allocations that emerge under segmentation with those that emerge when segmentation is not allowed, we see that the effect of segmentation on quality changes depending on the dispersion of preferences in the captive market – namely whether \( s > 1/2 \) or \( s < 1/2 \).

First, consider the case in which the captive market is relatively narrow, namely \( s < 1/2 \). Letting the subscript NS indicate the case where segmentation is disallowed and the subscript S indicate the case where segmentation is allowed, we have

\[
q_i^{NS} (z_i) \left\{ \begin{array}{lcl}
q_i^S (z_i) & \text{for} & z_i \leq s \\
< q_i^S (z_i) & \text{for} & z_i > s
\end{array} \right.
\]

Hence, segmentation has the effect of increasing the quality allocated to all \( z_i > s \) – while leaving that of \( z_i \leq s \) unchanged. As discussed in section 5.2, this occurs because the introduction of segmentation eliminates the discontinuity effect, and therefore makes less stringent the trade-off between efficiency and informational rents faced by the principal when selecting the quality allocated to \( z_i > s \).

Now consider the case in which the captive market is relatively wide, namely \( s > 1/2 \). In that case the equilibrium quality schedule exhibits pooling for all types in \([z_0, z_1]\) – where \( z_0 < 1/2 < z_1 \) – and

\[
q_i^{NS} (z_i) \left\{ \begin{array}{lcl}
q_i^S (z_i) & \text{for} & z_i \leq z_0 \\
> q_i^S (z_i) & \text{for} & z_i > z_0
\end{array} \right.
\]

Here, segmentation has the effect of decreasing the quality allocated to all \( z_i > z_0 \) – while leaving that of \( z_i \leq z_0 \) unchanged. Although segmentation eliminates the discontinuity effect – something
that, caeteris paribus, increases quality — it also eliminates the competition effect — something that, 
caeteris paribus, decreases quality. As shown in section 5.1, this latter element is dominant. The 
overall result is that, when \( s > 1/2 \), segmentation has a negative effect on quality, as it strictly 
lowers the quality offered to consumers with sufficiently low valuations.

7 Welfare comparison between segmented and non-segmented régimes

Proposition 4 (Effect of market segmentation on aggregate welfare.) When \( s > 1/2 \) disallowing 
market segmentation raises aggregate welfare, while when \( s < 1/2 \) it lowers it.

As proposition 4 indicates, a one-to-one correspondence exists between the effect of segmenta-
tion on equilibrium qualities, and its effect on aggregate welfare: segmentation increases welfare 
whenever it induces higher quality offers, and reduces it otherwise. This is especially intuitive 
when \( s < 1/2 \), since in that case disallowing segmentation lowers the quality offered to a subset 
of consumers (leaving the quality offered to the rest unchanged). By contrast, when \( s > 1/2 \), 
disallowing segmentation raises the qualities offered to some consumers. However, this does not 
necessarily mean that distortions are reduced. A subset of types may actually end up being offered 
qualities that are above their first-best levels. Hence, in this case the welfare effect of segmentation is 
unclear a priori. On the one hand, when segmentation is not allowed we obtain underprovision 
of quality (as we would in a monopoly). On the other hand, when segmentation is allowed, we may 
actually end up with quality overprovision for some consumers. Proposition 4 shows that from an 
aggregate welfare perspective this latter scenario is always preferable to the former.

An implication of proposition 4 is that, empirically, the welfare effect of segmentation can be 
inferrred by looking at the range of qualities offered by the firms. Consider \( s > 1/2 \). With 
segmentation, the lowest quality offered by each firm at equilibrium is \( v - 2z_i \), while without 
segmentation is above \( v - 2z_i \). The highest quality being offered by both firms is the same with 
and without segmentation, and is equal to \( v \). So, in this case, segmentation widens range of 
qualities being offered be the firms. By contrast, when \( s < 1/2 \) segmentation narrows the range 
of qualities being offered be the firms.\(^{24}\) Overall, therefore, our results suggest that segmentation 
 improves welfare whenever it narrows the range of qualities on offer, and it lowers welfare otherwise.

We now examine another issue of interest for policy-making when dealing with price segmenta-
tion, namely its effect on consumers located in different markets. The following proposition

\(^{24}\) With segmentation, the lowest quality offered by each firm is \( v - 1 \), while without segmentation it is below that 
value. The highest quality being offered by both firms is the same with and without segmentation, and is equal to \( v \).
addresses this issue.

**Proposition 5** *(Effect of market segmentation on consumer welfare).* *If market segmentation is allowed, then consumers located in the competitive market become better off, while captive consumers become worse off.*

As proposition 5 indicates, segmentation always harms captive consumers, while it makes consumers located in the competitive market better off. This shares some similarities with the results obtained by the literature on third-degree price discrimination within the context of a monopoly – such as e.g. Varian (1985) – where discrimination redistributes income away from “low elasticity” consumers towards “high elasticity” consumers and the producer.

The rationale for the result is simple. First, consider captive consumers. It is easy to see that they always benefit from disallowing segmentation. In the competitive market, competition induces the firms to relinquish higher rents to consumers. If the firms cannot discriminate, captive consumers also benefit from this. Those with $z_i \leq 1/2$ are offered the same contracts that are also offered to consumers in the competitive market, and are therefore directly affected by competition there. Captive consumers with $z_i > 1/2$ (if any) do not enjoy such direct benefits, since no consumers in the competitive market share their low valuations. However, they nonetheless benefit, through the competition effect highlighted in section 5.

Now consider consumers located in the competitive market. Disallowing segmentation makes the firms more reluctant to compete against each other by raising consumer rents. This is because those higher rents also need to be offered to captive consumers.\(^{25}\) As a result, without segmentation, the equilibrium value of $u(1/2)$ is lower than with segmentation, something that hurts the consumers.\(^{26}\)

What about quality allocations? From the incentive compatibility constraint IC.1, $u_i(z_i) = u(1/2) + \int_{z_i}^{1/2} q_i(x)dx$. For a given value of $u(1/2)$, rents in the competitive market are clearly affected by qualities allocations in that market. Through the discontinuity effect, when captive markets are narrow, segmentation raises the qualities offered to consumers with $z_i \in (s, 1/2]$.

So here segmentation benefits the consumers in the competitive market in two ways: it raises $u(1/2)$ and it also raises the qualities offered to some of them. When captive markets are wide, however, segmentation lowers the qualities offered to consumers with $z_i > z_0$. We have two contrasting effects. On one hand, segmentation raises $u(1/2)$, while on the other it lowers the qualities offered to some consumers. Proposition 5 shows that the former effect is always dominant.

\(^{25}\)This shares similarities with example 2 in Galera and Zaratiegui (2006).

\(^{26}\)This is formally proved in the appendix (proof of proposition 5).
8 Concluding remarks

To our knowledge, this paper is the first to explicitly analyze the effects of market segmentation – namely, discrimination between captive and non-captive consumers – in a setting where firms compete by offering non-linear price-quality contracts. We identify two effects of disallowing segmentation, a discontinuity effect and a competition effect, working in opposite directions. The overall impact of segmentation on quality allocations – and welfare – depends on the interplay between these two effects. In turn, this depends on whether the captive market is “narrow” or “wide”.

In this paper, captive markets are exogenously given. In reality, however, firms may take actions that affect their captive consumers base. Future research could analyze the two-stage game where firms first select their captive base, and then compete by offering price-quality menus. This would also open up the possibility of asymmetric environments/equilibria. For instance, first-movers may invest in creating a large base of captive consumers, in order to obtain a dominant position (this is what Fudenberg and Tirole 1984 call a “top dog” strategy). Although in the present paper we concentrate on symmetric environments and symmetric equilibria (primarily for analytical convenience), the qualitative effects we identify are independent of the precise details of the model (such as for instance whether there is full symmetry between the firms). Our analysis therefore provides a useful building block for further research on the topic.

9 Appendix

Proof of proposition 1

Captive market The problem faced by the principal is a standard screening model, à la Mussa and Rosen (1978). Firm i’s optimal contractual offer to consumers located in the competitive market solves (2) subject to (IC.1), (IC.2) and participation: \( u_i(z_i) \geq 0 \) \( \forall z_i \in [0, s] \). Given (IC.1), the necessary and sufficient condition for participation is that \( u_i(s) \geq 0 \). Ignoring constraint (IC.2) for the time being, we substitute for \( u_i(z_i) \) using (IC.1) and operate integration by parts. The firm’s problem can then be written as

\[
\max_{q_i(z_i)} \int_0^s \frac{1}{3s} \left( q_i(z_i)(v - 2z_i) - \frac{q_i(z_i)^2}{2}\right) dz_i.
\]

It is straightforward to show that the unconstrained optimal quality allocations are given by (3) (and therefore satisfy (IC.2)).

\[27\text{For instance, firms may incur expenditures that make switching costly for at least some of their consumers (as in Tirole 1988, p. 326).}\]
The Lagrangian for the problem is then

\[
\max_{q_i(z_i), z_i^m} \int_0^{z_i^m} \frac{1}{3} \left( q_i(z_i)(v - 2z_i) - \frac{q_i(z_i)^2}{2} \right) dz_i - \frac{1}{3} B_i(z_i^m) z_i^m.
\]

The first order condition with respect to \(z_i^m\) is

\[
q_i(z_i^m)(v - 2z_i^m) - \frac{q_i(z_i^m)^2}{2} - B_i'(z_i^m) z_i^m - B_i(z_i^m) = 0.
\]

Substituting for \(z_i^m = 1/2\), \(q_i(z_i^m) = v - 2z_i^m = v - 1\) and allowing for symmetry, we obtain \(u_i(1/2) = (v - 1)(v - 2)/2\), \(i = L, R\).

**Proof of proposition 2**

As seen in the main text, when \(s > 1/2\) firm \(i\)’s equilibrium contractual offer must solve (11), subject to (13), the captive consumer’s participation constraint, where \(z_i^m\) is defined by \(u_i(z_i^m) = B_i(z_i^m)\). Condition (14) is necessary and sufficient condition for (13) to hold. Ignore constraint (IC.2) for the time being. The Lagrangian for the problem is then

\[
L = \max_{q_i(z_i), z_i^m} \int_0^{z_i^m} \frac{1}{3s} \left( (v - z_i) q_i(z_i) - \frac{q_i(z_i)^2}{2} + \int_{z_i}^{z_i^m} q_i(x) dx \right) dz_i + \int_{z_i^m}^{s} \frac{1}{3s} \left( (v - z_i - 3s\lambda) q_i(z_i) - \frac{q_i(z_i)^2}{2} + \int_{z_i^m}^{z_i} q_i(x) dx \right) dz_i - \frac{B_i(z_i^m)}{3}(z_i^m + 1 - 3\lambda)
\]

where \(\lambda \geq 0\) is the Lagrange multiplier for constraint (13). After integration by parts, this becomes (15). The FOC for \(z_i^m\) is

\[
\frac{1}{3s} \left( (v - 2k_i^m)q_i(k_i^m) - \frac{q_i(k_i^m)^2}{2} \right) - \frac{1}{3s} \left( (v - 2z_i^m + s - 3s\lambda)q_i^+(z_i^m) - \frac{q_i^+(z_i^m)^2}{2} \right) = - \frac{B_i'(z_i^m)}{3}(z_i^m + 1 - 3\lambda) - \frac{B_i(k_i^m)}{3} = 0
\]

where \(q_i^+(k_i^m) = \lim_{\varepsilon \to 0} q_i(z_i^m + \varepsilon)\). The first order condition with respect to \(\lambda\) is

\[
\lambda u_i(s) = 0.
\]

With interior solutions, the unconstrained equilibrium quality allocations are given by (16) and (17). These violate (IC.2) whenever \(\lambda < 1/3\). In that case, the second-best quality schedule must prescribe pooling over some range \([z_0, z_1]\), where \(0 \leq z_0 < 1/2 < z_1 \leq s\). As shown e.g. by Laffont and Martimort (2002, chapter 3), the optimal pooling procedure must satisfy (20). There are three

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28 It is straightforward to show that the second order condition for \(z_i^m\) is always satisfied under (IC.2).

29 The distinction between \(q_i(z_i^m)\) and \(\lim_{\varepsilon \to 0} q_i(z_i^m + \varepsilon)\) is meaningful because, as shown below, in the absence of pooling the optimal quality schedule may exhibit a jump at \(z_i^m\).
possible scenarios: (i) pooling binds only on the left: \( q = v - 2z_0 \) for some 0 ≤ \( z_0 \) < 1/2, and \( q < v - 2z_1 + s(1 - 3\lambda) \) for all \( s \geq z_1 > 1/2 \); (ii) pooling binds only on the right: \( q = v - 2z_1 + s(1 - 3\lambda) \) for some \( s \geq z_1 > 1/2 \), and \( q > v - 2z_0 \) for all 0 ≤ \( z_0 \) < 1/2; (iii) pooling binds on both sides: \( q = v - 2z_0 \) for some 0 ≤ \( z_0 \) < 1/2, and \( q = v - 2z_1 + s(1 - 3\lambda) \) for some \( s \geq z_1 > 1/2 \). Note however that since \( z_1 \geq 1/2 \), \( \lambda \geq 0 \) and \( s \leq 1 \), \( q \leq v \) follows. Hence, at equilibrium, case (ii) cannot arise; we must have \( q = v - 2z_0 \) for some 0 ≤ \( z_0 \) < 1/2.

We now show that under A1 no equilibria without pooling can exist. This is because \( z_i^m = 1/2 \) is inconsistent with \( \lambda \geq 1/3 \). Substituting for \( q_i(z_i^m) \), \( q_i^+(z_i^m) \) (from (16) and (17)), \( z_i^m = 1/2 \) and allowing for \( B_i(z_i^m) = u_i(z_i^m) \) (for symmetry), (23) can be written as

\[
\frac{1 + s}{2s} (v - 1)^2 - \frac{1}{2s} \max\{(v - 2z_1 + s - 3s\lambda), 0\}^2 - 3(v - 1)(1/2 - \lambda) - u_i(1/2) = 0 \tag{25}
\]

We consider three cases

Case 1: \( \lambda < (v - s)/3s \), so that \( v - 2z_1 + s - 3s\lambda > 0 \) for all \( z_i \leq s \). In this case, (25) can be rewritten as \( \frac{1 + s}{2s} (v - 1)^2 - \frac{1}{2s} (v - 1 + s - 3s\lambda)^2 - 3(v - 1)(1/2 - \lambda) - \int_{1/2}^s (v - 2z_1 + s - 3s\lambda)dz_i = 0 \). Under A1 the values of \( \lambda \) that solve this equation are inconsistent with \( (v - s)/3s > \lambda \geq 1/3 \).

Case 2: There exists a \( s \) satisfying \( s > \hat{s} > 0.5 \) such that \( v - 2\hat{s} + s - 3s\lambda = 0 \). Notice that a necessary condition for this to hold is that \( \lambda < (v - 1 + s)/3s \). In this case, (25) becomes \( \frac{1 + s}{2s} (v - 1)^2 - \frac{1}{2s} (v - 1 + s - 3s\lambda)^2 - 3(v - 1)(1/2 - \lambda) - \int_{1/2}^s (v - 1 + s - 3s\lambda)dz_i = 0 \). Under A1 the values of \( \lambda \) that solve this equation are inconsistent with \( (v - 1 + s)/3s > \lambda \geq 1/3 \).

Case 3: \( \lambda \geq (v - 1 + s)/3s \), so that \( u_i(0.5) = 0 \). In this case, (25) becomes \( \frac{1 + s}{2s} (v - 1)^2 - 3(v - 1)(1/2 - \lambda) = 0 \). Under A1 the values of \( \lambda \) that solve this equation are inconsistent with \( \lambda \geq (v - 1 + s)/3s \).

This proves that under A1 there are no equilibria without pooling. Hence, at equilibrium \( \lambda < 1/3 \), and pooling emerges over some interval \([z_0, z_1]\). The quality allocations in \([z_0, s]\) are therefore above those that would be offered by a monopolist. Finally, the equilibrium value of \( u_i(1/2) \), \( i = L, R \) is found by imposing symmetry on (23).

**Proof of proposition 3**

When \( s < 1/2 \) firm \( i \)'s equilibrium contractual offer must satisfy

\[
\max_{q_i(z_i), z_i^m} \int_0^s \frac{1 + s}{3s} \left( (v - z_i) q_i(z_i) - \frac{q_i(z_i)^2}{2} - u_i(z_i) \right) dz_i + \int_{z_i^m}^{z_i^m} \frac{1}{3} \left( (v - z_i) q_i(z_i) - \frac{q_i(z_i)^2}{2} - u_i(z_i) \right) dk_i
\]

subject to (IC.1) and (IC.2). Ignore constraint (IC.2) for the time being. Substituting for \( u_i(z_i) \)

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30 More precisely, \( u_i(1/2) = u_i(s) + \int_{1/2}^s [d_1(2s)\max\{(v - 2z_1 + s - 3s\lambda), 0\}]dz_i \). However, from (24), when \( \lambda \geq 1/3 > 0 \) then necessarily we must have \( u_i(s) = 0 \).

31 This ensures that \( v - 2z_1 + s - 3s\lambda > 0 \) for \( z_1 = 1/2 \).

32 The captive consumer’s participation constraint, (13), is here automatically satisfied, since \( u_i(s) > u_i(z_i^m) = B_i(z_i^m) \geq 0 \).
from (10), and after integration by parts, the problem becomes
\[
\max_{q_i(z_i),2} \int_0^s \frac{1}{3s} \left( (v-2z_i) q_i(z_i) - \frac{q_i(z_i)^2}{2} \right) dz_i + \\
\int_s^{z_i^m} \frac{1}{3} \left( (v-2z_i - 1) q_i(z_i) - \frac{q_i(z_i)^2}{2} \right) dk_i - \frac{B_i(z_i^m)(1+z_i^m)}{3}.
\]

The unconstrained equilibrium quality allocations satisfy (21) and (22).\(^{33}\) The first order condition\(^{34}\) with respect to \(z_i^m\) is
\[
q_i(z_i^m)(v-2z_i^m - 1) - \frac{q_i(z_i^m)^2}{2} - B_i(z_i^m)(1+z_i^m) - B_i(z_i^m) = 0. \tag{26}
\]

Allowing for symmetry, we can rewrite (26) as
\[
q_i(z_i^m)(v-2z_i^m - 1) - \frac{q_i(z_i^m)^2}{2} - q_i(z_i^m)(1+z_i^m) - u_i(z_i^m) = 0.
\]

Substituting for \(q_i(z_i^m) = v-2z_i^m - 1, z_i^m = 1/2\) we obtain \(u_i(1/2) = (v-2)(v-5)/2\). If \((v-2)(v-5)/2 > 0\), this is the equilibrium value of \(u_i(1/2)\). Otherwise, \(u_i(1/2) = 0.\(^{35}\)

**Proof of proposition 4**

**Part 1:** When \(s > 1/2\), aggregate welfare is higher when segmentation is not allowed.

(i) First, consider \(z_i \in [z_1, s]\) (if any). For these types, the equilibrium quality allocation without segmentation is \(q_i^{NS}(z_i) = v-2z_i + s - 3s\lambda\), where \(\lambda < 1/3\), while that with segmentation is given by \(q_i^S(z_i) = v-2z_i < q_i^{NS}(z_i)\). The difference between surplus without segmentation and that with segmentation is
\[
(v-z_i) q_i^{NS}(z_i) - \frac{q_i^{NS}(z_i)^2}{2} - (v-z_i) q_i^S(z_i) + \frac{q_i^S(z_i)^2}{2}. \tag{27}
\]

Substituting for \(q_i^{NS}(z_i)\) and \(q_i^S(z_i)\) in (27) we obtain \(\frac{1}{2} s (1-3\lambda)(-s+2z_i+3s\lambda) > 0\), where the last inequality follows from \(s \leq 1, z_i \geq 1/2, \lambda \geq 0\). (ii) Second, consider \(z_i \in (z_0, z_1)\). For these types \(q_i^{NS}(z_i) = \bar{q} = v-2z_0\), while \(q_i^S(z_i) = v-2z_i < \bar{q}\). Substituting for these values in (27), we obtain \(2z_0(z_i - z_0) > 0\). (iii) Finally, for \(z_i \in [0, z_0]\) (if any), the equilibrium quality allocation with and without segmentation are equal. Surplus is therefore also equal.

**Part 2:** When \(s < 1/2\), aggregate welfare is higher when market segmentation is allowed.

From inspection of (21),(22) and (3), \(q_i^{FB}(z_i) \geq q_i^S(z_i) \geq q_i^{NS}(z_i)\) (the last inequality being strict for some \(z_i\)). Hence, aggregate welfare is higher under segmentation.\(^{\blacksquare}\)

**Proof of proposition 5**

\(^{33}\) It is easy to verify that under A1 these allocations are strictly positive and satisfy the monotonicity constraint (IC.2).

\(^{34}\) It is easily checked that the second order condition for \(z_i^m\) is always satisfied.

\(^{35}\) Note that, when \(B_i(z_i) = 0\) for all \(z_i \leq 1/2\), it cannot be optimal for firm \(i\) to set \(z_i^m < 1/2\). If \(z_i^m < 1/2\), the lhs of (26) becomes \(q_i(z_i^m)(v-2z_i^m - 1) - \frac{q_i(z_i^m)^2}{2} > 0\). Hence, the market is fully covered.
Part 1): Consumers located in the competitive market are better off when segmentation is allowed.

i) $s > 1/2$. Without segmentation utility is

$$u^{NS}(z_i) = \begin{cases} 
  u^{NS}(1/2) + \varphi(1/2 - z_0) + \int_{z_i}^{z_0} (v - 2x) \, dx & \text{if } z_i < z_0 \\
  u^{NS}(1/2) + \varphi(1/2 - z_i) & \text{if } 1/2 \geq z_i \geq z_0 
\end{cases}$$

where $1/2 \geq z_i$ since we are considering consumers in the competitive market. From proposition 2, $u^{NS}(1/2) = \varphi(v + 6\lambda - 3.5) - 0.5\varphi^2$, and $\varphi = v - 2z_0$. In contrast, with segmentation utility is

$$u^{S}(z_i) = u^{S}(1/2) + \int_{z_i}^{1/2} (v - 2x) \, dx$$

where (from proposition 1) $u^{S}(1/2) = (v - 1) (v - 2)/2$. Clearly, $u^{NS}(z_i) - u^{S}(z_i)$ is greatest for $z_i < z_0$, where it is equal to

$$u^{NS}(1/2) - u^{S}(1/2) + \varphi(1/2 - z_0) - \int_{z_0}^{1/2} (v - 2x) \, dx. \quad (29)$$

Substituting for $\varphi = v - 2z_0$, (29) becomes $-2v + 6z_0 = 6v\lambda - 12z_0\lambda - v_0 - 0.75$, which is negative for all $\lambda$ consistent with pooling (namely $1/3 > \lambda \geq 0$) and all $z_0 \geq 0$.36

ii) $s < 1/2$. Without segmentation, utility is

$$u^{NS}(z_i) = \begin{cases} 
  u^{NS}(1/2) + \int_{z_i}^{1/2} (v - 2x) \, dx & \text{if } z_i < s \\
  u^{NS}(1/2) + \int_{s}^{1/2} (v - 2x - 1) \, dx - \int_{z_i}^{s} (v - 2x) \, dx & \text{if } z_i \geq s
\end{cases}$$

where, from proposition 3, $u^{NS}(1/2) = \max\{0, (v - 2) (v - 5)/2\}$. With segmentation, utility is given by (28). Since $u^{NS}(1/2) - u^{S}(1/2) = \max\{- (v - 1) (v - 2)/2, 1 - v/2\} < 0$ under A1, $u^{NS}(z_i) - u^{S}(z_i) < 0$ follows.

Part 2): Captive consumers are better off when segmentation is not allowed

This follows because disallowing segmentation (i) (weakly) raises the quality levels offered to all captive consumers (and strictly raises the qualities offered to some of them) and (ii) weakly increases $u(s)$. Under (IC.1), both (i) and (ii) raise the captive consumers’ welfare. ■

10 References


36Note also that $u^{NS}(1/2) - u^{S}(1/2) = (v - 2z_0) (v + 6\lambda - 3.5) - 0.5 (v - 2z_0)^2 - 0.5(v - 1) (v - 2)$. This is negative for all $0 \leq z_0 \leq 1/2$. Hence, $u^{NS}(1/2) < u^{S}(1/2)$, as argued in the main text.


