

# Fiscal Multiplier in a Credit-Constrained New Keynesian Economy\*

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## Abstract

Using a dynamic stochastic general equilibrium (DSGE) model that accounts for credit constraints, we study the effects of fiscal stimulus on the macroeconomy. We show that the presence of credit constraints results in larger fiscal multipliers than indicated by the standard DSGE models. If credit-crunch conditions persist, the multipliers become large enough for fiscal policy to be highly effective.

Keywords: DSGE models, Monetary Policy, Fiscal Policy, Liquidity Trap, Credit constraints.

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# 1 Introduction

Over the last decade, in many if not all developed countries (including the Euro-Area, the US and the UK), monetary policy has been the main instrument for managing the level and the rate of growth of aggregate demand and inflationary pressures. The chief monetary policy tool has been short term interest rates. The response to the recent financial crisis has typically been to lower the short term nominal interest rate to its zero lower bound (i.e. generating a liquidity trap). At the zero lower bound, monetary policy loses its power, meaning that the conventional policy option of reducing interest rate is no longer available.

The ineffectiveness of monetary policy at the zero lower bound has brought back an old question: can fiscal policy be used to stimulate economic activities? Several recent papers have sought an answer to this question. The majority of empirical research in this area seems to suggest that fiscal policy is not an effective policy and that an increase in government spending does not have a significant impact on the economy (see, e.g., Ramey (2011), Hall (2009) and references therein). The fiscal multiplier is typically estimated to lie between 0.5 and 1. Findings based on model-based (or theoretical) analysis are in line with the results of empirical research. Most of the theoretical discussion of this issue has been based on the state-of-the-art instances of New Keynesian economics (e.g. Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007); see Cogen, Cwik, Taylor and Wieland (2010) for a survey). The observation that the fiscal multiplier is small in standard New Keynesian models is perhaps unsurprising. The New Keynesian models are based on frictionless Real Business Cycles models predicated upon perfectly functioning financial markets. Under these conditions, an increase in government expenditure would crowd out private spending, leading to an increase in the real interest rate. This finding opposes with the traditional Keynesian view that an increase in government expenditure has an multiplier effect on output, since it increases households' disposable income and

consumption expenditures.

However, recent works such as those by Christiano, Eichenbaum and Rebelo (2011) and Woodford (2011) show that the conclusion that arises from New Keynesian models - in brief, that fiscal policy is ineffective - changes if the economy stays at the zero lower bound for a prolonged period of time and fiscal expansion lasts exactly as long as the zero-bound state. These studies report a fiscal multiplier considerably in excess of one. In one version of their model, Christiano, Eichenbaum and Rebelo (2011) report a multiplier as high as 3.7. The explanation for this result is that an increase in government expenditure increases output which in turn leads to a rise in inflation. When the interest rate is stuck at zero, increased inflation reduces the real interest rate, leading to a further increase in output. In these studies, however, the duration of the liquidity trap is exogenously determined. Such an approach may exaggerate the fiscal multiplier. One would expect effective fiscal policy help push the economy out of a liquidity trap. If so, the multiplier would be smaller (see Erceg and Linde (2012) for further discussion). In addition, the condition that the fiscal expansion should last exactly as long as the zero-bound state is highly restrictive, given the possibility of bureaucratic delays.

In this paper, we examine the question of the usefulness of government spending as a means of stimulating the economy within a model in which the liquidity trap arises from the model itself. To achieve this, we employ the model proposed by Del Negro, Eggertsson, Ferrero and Kiyotaki (2011) (henceforth “DEFK”), which offers an explanation for the 2008 financial crisis. In this otherwise standard New Keynesian model, the agents are credit-constrained by borrowing constraints as well as a resaleability constraint on their asset holdings. Entrepreneurs invest in capital. They can borrow money to invest by issuing equity. However, there is a maximum limit on the amount of equity that entrepreneurs can issue in a given period. In other words, they face borrowing constraints. Households can save by purchasing government

bonds or private equity issued by others. Compared with government bonds, private equity is illiquid in the sense that households can sell only up to a certain portion of their equity holdings in a given period. During a credit crisis, the resaleability of private equity drops, further restricting households' liquidity. This leads to a substantial drop in both output and inflation. The central bank, who follows the Taylor rule, aggressively lowers the nominal interest rate to its zero lower bound. The model assumes that during such times, the central bank implements quantitative easing through the purchase of private equity in the open market. A comparison of empirical data and the model's simulations shows that this model performs well in explaining the responses of the key macro variables to the recent credit crisis. Thus, the model captures the essence of the recent crisis as well as the central banks' responses to the crisis.

Our first contribution to the model is the introduction of a role for government spending. Our second contribution is methodological. DEFK, following the standard practice in the literature, approximate the model's equations by log-linearising them around the steady state. Since in this paper we consider crisis situations in which the economy may spend a long time away from the steady state, we simulate numerically the original non-linear model. In the latter case, the accuracy of results does not depend on the economy's remaining in the immediate vicinity of the steady state.

We find that the presence of credit constraints results in a multiplier of around one, which is much greater than that suggested by a standard DSGE model. The multiplier increases substantially if credit-crunch conditions persist. This is true even when fiscal expansion exceeds the duration of the liquidity trap.

The remainder of the paper is structured as follows: Section 2 describes the special features of the DEFK model; Section 3 compares the values of the government spending multipliers produced by different models in normal times and in times of crisis; and Section 4 concludes.

## 2 The Model with Credit Frictions

This section describes the model that we use in our analysis. Developed by DEFK (2011), the model features a credit-constrained economy in which households face random shocks that tighten their liquidity constraints. Government expenditure is absent in the original DEFK model. We introduce it to the model so that we can use it to study the effects of fiscal stimulus on the macroeconomy.

The DEFK model incorporates a specific form of credit frictions as suggested by Kiyotaki and Moore (2008). As in a standard New Keynesian DSGE model, the major participants in the DEFK model are households, labour agencies, intermediate- and final-goods producers, capital producers and the government. A unique feature of the model is that households consist of both entrepreneurs and workers. As noted above, households face credit constraints. The government can implement quantitative easing through the purchase of private equity in the open market. Other aspects of the model are standard New Keynesian (see, for example, Smets and Wouters (2007) (hereafter “SW”)). Households choose the amounts of consumption and labour in each period to maximise their discounted utility over an infinite life horizon. Household members who work supply differentiated labour to the production sector through the arrangement of employment agencies, who bundle differentiated labour supply into homogeneous units for firms to hire. Wages are negotiated by labour unions representing each specific type of workers. Labour unions enjoy some degree of monopoly power which allows them to set wages according to the Calvo-pricing scheme. Intermediate-goods producers choose the optimal amounts of labour and capital inputs that maximise their expected profits, taking wages and rental rate of capital as given. They set prices for their differentiated products based on the Calvo-pricing assumption, and sell them to final-goods producers for the production of homogeneous final goods. Capital-goods producers convert final goods into physical capital, which incurs an adjustment cost. The government conducts

monetary policy according to a Taylor-style rule. We describe in the following sections the unique features of the DEFK model, and leave the standard parts to the appendix.

## 2.1 Credit-Constrained Households

The economy consists of a continuum of identical households. Each household consists of a continuum of members  $j \in [0, 1]$ . In each period, members of the household have an i.i.d. opportunity  $\varkappa$  to invest in capital. Household members  $j \in [0, \varkappa)$  who receive the opportunity to invest are referred to as “entrepreneurs”. Household members  $j \in [\varkappa, 1]$  who do not receive investment opportunity are “workers”. Entrepreneurs invest and do not work. Workers supply differentiated labour to earn labour income.

The model assumes that all the assets of a household are shared equally among its members. At the beginning of each period, each household member gets an equal share of the household’s assets. After members find out whether they are entrepreneurs or workers, the household cannot redistribute its assets. If any members of the household need additional funds during the period, they need to obtain them from external sources. This assumption is important as it gives rise to liquidity constraints. At the end of the period, household members return all their income and assets to the pool which will then be divided by all household members at the beginning of next period.

The objective of each household member is to maximise the utility of the household as a whole. The household’s utility at  $t$  depends on the aggregate amount of consumption goods  $C_t \equiv \int_0^1 C_t(j) dj$  bought by its members because of the assumption that consumption goods are shared among household members. All members therefore face the same utility function which is given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\sigma}}{1-\sigma} - \frac{1}{1+v} \int_{\varkappa}^1 H_s(j)^{1+v} dj \right], \quad (1)$$

where  $\beta \in (0, 1)$  is the discount factor,  $\sigma > 0$  is the coefficient of relative risk aversion, and  $v$  is the inverse Frisch elasticity of labour supply. Labour supply  $H_t(j) = 0$  for entrepreneurs  $j \in [0, \varkappa)$ . Each period, household members choose optimally among purchases of non-durable consumption goods, savings in bonds or equity and, if they are entrepreneurs, investment in capital. Details of their saving and investment options are as follows: (i) *Investment in physical capital (applicable to entrepreneurs only)*: Investment in new capital  $I_t$  costs  $p_t^I$  per unit. Each unit of capital stocks generates a rental income of  $r_t^k$  and depreciates at a rate  $\delta$ . Capital can be traded at a market value of  $q_t$  per unit. So the return on investment in new capital over  $t$  to  $t + 1$  is  $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{p_t^I}$ . Entrepreneurs can borrow to invest. Borrowing is in the form of issuing equity  $N_t^I$  which entitles its holders to claims to the future returns on the capital. (ii) *Saving by buying private equity*: Household members can buy the equity  $N_t^O$  issued by other households at the market price  $q_t$ . Equity pays its holder the future returns on the pledged capital, therefore the return on buying equity over  $t$  to  $t + 1$  is  $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t}$ .<sup>1</sup> (iii) *Saving by buying government bonds*: Alternatively, household members can save by buying risk-free government bonds,  $L_t$ , which have a unit face value and pay a gross nominal interest rate  $R_t$  over the period  $t$  to  $t + 1$ . The net equity holding  $N_t$  of a household is defined as the sum of their capital stocks and their holdings of others' equity minus any equity issued:

$$N_t = N_t^O + K_t - N_t^I \quad (2)$$

At the beginning of each period, households also receive dividends from intermediate-goods and capital-goods firms amounting to  $D_t = \int_0^1 D_t(i) di$  and  $D_t^K$  respectively. In addition, households pay lump-sum taxes  $\tau_t$  to the government. Taxes are lump-sum so they do not affect the labour supply and investment/saving decisions of household members. The household's

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<sup>1</sup>The implicit assumption is that holding the equity issued by other households has the same risk level as holding capital directly.

intertemporal budget constraint is:<sup>2</sup>

$$\begin{aligned}
& C_t + p_t^I I_t + q_t [N_t - I_t] + L_t \\
= & [r_t^k + (1 - \delta) q_t] N_{t-1} + \frac{R_{t-1} L_{t-1}}{\pi_t} + \int_{\infty}^1 \frac{W_t(j)}{P_t} H_t(j) dj \\
& + D_t + D_t^K - \tau_t
\end{aligned} \tag{3}$$

where  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation rate at  $t$  and  $W_t(j)$  is the nominal wage earned by type- $j$  workers. Entrepreneurs and workers face different decision problems as explained below:

### 2.1.1 Entrepreneurs

In the steady-state and the post-shock equilibria in our simulations, the market price of equity  $q_t$  is always greater than the cost of new capital  $p_t^I$ . Therefore, the return on investment in new capital  $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{p_t^I}$  is strictly greater than the return on buying equity  $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t}$  which is the same as the real return on government bonds due to the anti-arbitrage condition. A rational entrepreneur will use all the available resources to fund their investment in new capital. Entrepreneurs face constraints in obtaining funds. These constraints exist in two forms: (i) *Borrowing constraint*: Entrepreneurs can only borrow funds by issuing equity of up to  $\theta \in (0, 1)$  fraction of their new investment; (ii) *Resaleability constraint*: In each period, entrepreneurs can only sell a maximum of  $\phi_t \in (0, 1)$  fraction of their net equity holdings. The amount of liquidity in the economy is increasing in  $\theta$  and  $\phi_t$ . When the values of  $\theta$  and  $\phi_t$  are low, it is more difficult for entrepreneurs to obtain funds for their investment, i.e., their credit constraints tighten. Liquidity shocks, as detailed later, are modelled as sudden drops in  $\phi_t$ . From entrepreneurs' first-order conditions for  $C_t(j)$ ,  $L(j)$  and  $N_t(j)$ , we obtain the aggregate in-

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<sup>2</sup>In this paper, stock variables at  $t$  show the amounts at the *end* of the period. This is different from the timing convention of stock variables in DEFK. In their paper, they define stock variables at  $t$  as the amounts at the *beginning* of the period.

vestment function: (see appendix for details)

$$I_t = \int_0^{\varkappa} I_t(j) dj = \varkappa \frac{[r_t^k + (1 - \delta) q_t \phi_t] N_{t-1} + \frac{R_{t-1} L_{t-1}}{\pi_t} + D_t + D_t^K - \tau_t}{p_t^I - \theta q_t} \quad (4)$$

This investment function is different from the one in a standard DSGE model that assumes a perfect capital market. It can be seen from this equation that aggregate investment expenditure falls when the credit constraints tighten.

### 2.1.2 Workers

The workers' consumption/saving decisions can be solved by considering the household as a whole. The household's problem is to choose  $C_t$ ,  $L_t$  and  $N_t$  to maximise utility (1) subject to its intertemporal budget constraint (3) and financing constraint on investment (4). The first-order conditions give the respective Euler equations for bonds and equity:

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{R_t}{\pi_{t+1}} + \frac{\varkappa (q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \right\} \quad (5)$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{r_{t+1}^k + (1 - \delta) q_{t+1}}{q_t} + \frac{\varkappa (q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{r_{t+1}^k + (1 - \delta) q_{t+1} \phi_{t+1}}{q_t} \right] \right\} \quad (6)$$

By contrast, the Euler equations in a standard DSGE model without credit frictions would be:

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left( \frac{R_t}{\pi_{t+1}} \right) \right\}$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{r_{t+1}^k + (1 - \delta) q_{t+1}}{q_t} \right] \right\}$$

The DEFK model suggests there is a premium on top of the standard returns on bonds and equity. In equation (5), the term  $\frac{\varkappa(q_{t+1}-p_{t+1}^I)}{p_{t+1}^I-\theta q_{t+1}} \cdot \frac{R_t}{\pi_{t+1}}$  represents the premium a bond-holder enjoys. This premium arises due to the fact that entrepreneurs are credit-constrained. By putting their money into one extra unit of bonds at  $t$  (instead of spending on consumption goods), they can earn  $\frac{R_t}{\pi_{t+1}}$  extra units of liquidity at  $t+1$ . This extra liquidity allow them to profit from the investment opportunity given that it arrives at  $t+1$ . Similarly, equation (6) shows that there is a premium enjoyed by an equity-holder. By choosing to invest in one extra unit of equity at  $t$  instead of spending, the equity holder receives  $\frac{r_{t+1}^k+(1-\delta)q_{t+1}\phi_{t+1}}{q_t}$  extra units of liquidity at  $t+1$ . The resaleability-constraint parameter  $\phi_{t+1}$  appears in this term because one can only sell a maximum portion  $\phi_{t+1}$  of their equity holding at  $t+1$ . The workers' wage-setting and the firms' price-setting decisions are standard in the DEFK model. We include the details together with the first-order conditions in the appendix.

## 2.2 Government Policies

One of the innovations in the DEFK model is that the government carries out quantitative easing in the event of a credit crisis. During a credit crisis (represented by a negative shock to the resaleability constraint parameter  $\phi_t$ ), the government buys equities  $N_t^g$  from households by selling bonds  $L_t$ . Unlike private equity, government bonds are not subject to resaleability constraint and hence not affected by the crisis. Households' liquidity increases as a result of quantitative easing. The amount of private equity bought by the government is proportional to the magnitude of the credit shock:

$$\frac{N_t^g}{K} = \psi_k \left( \frac{\phi_t}{\phi} - 1 \right), \quad (7)$$

where  $\phi$  is the steady-state value of  $\phi_t$  and  $\psi_k < 0$  is the policy parameter.

In DEFK's original work, there is no government expenditure on con-

sumption and investment goods. We add exogenous government expenditure,  $G_t$ , to their model so that we can use it to study the fiscal multiplier. The government's budget constraint is:

$$G_t + q_t N_t^g + \frac{R_{t-1} L_{t-1}}{\pi_t} = \tau_t + [r_t^k + (1 - \delta) q_t] N_{t-1}^g + L_t \quad (8)$$

It implies that the government has to finance its consumption spending  $G_t$ , equity purchases  $q_t N_t^g$  and debt repayments  $\frac{R_{t-1} L_{t-1}}{\pi_t}$  by lump-sum taxes  $\tau_t$ , returns on its equity holdings  $[r_t^k + (1 - \delta) q_t] N_{t-1}^g$  and income from bond sales  $L_t$ . The government imposes a taxation rule such that the tax income at period  $t$  is proportional to its net liability at the beginning of the period:

$$\tau_t - \tau = \psi_\tau \left[ \left( \frac{R_{t-1} L_{t-1}}{\pi_t} - \frac{RL}{\pi} \right) - q_t N_{t-1}^g \right], \quad (9)$$

where  $\psi_\tau > 0$ .  $\tau$  and  $\frac{RL}{\pi}$  are the respective steady-state values of tax and government's debt position.  $q_t N_{t-1}^g$  represents the value of government's equity holdings at the beginning of the period. The steady-state value of  $N^g$  is zero by assumption. As the adjustment on taxes is slow compared to bond issue (reflected by a low  $\psi_\tau$ ), the government has to obtain funds for an open market intervention mainly by issuing bonds.

For the monetary policy rule, we do not follow DEFK to use a strictly inflation-targetting rule. Instead we refer to a generalised Taylor rule similar to the one in SW which targets both inflation and the output gap. The nominal interest rate also follows a short-run feedback from the change in the output gap:

$$R_t = \max \left\{ R_{t-1}^{\rho_R} \left( R \pi_t^{\psi_\pi} \left( \frac{Y_t}{Y} \right)^{\psi_Y} \right)^{1-\rho_R} \left( \frac{Y_t}{Y_{t-1}} \right)^{\psi_{\Delta Y}}, 1 \right\} \quad (10)$$

where  $R_t$  is the gross nominal interest rate,  $R$  and  $Y$  are the natural gross nominal interest rate and the natural output respectively,  $\rho_R$  captures the degree of interest rate smoothing,  $\psi_\pi > 1$  and both  $\psi_Y$  and  $\psi_{\Delta Y}$  are between

zero and one. The zero lower bound on the nominal interest rate requires that  $R_t$  cannot be lower than 1.

### 2.3 Calibration

Most of the calibration in this paper is drawn from the estimations of SW, except for the parameters related to credit frictions which largely follow DEFK. The calibrated values of parameters are summarised in Table 1. Two important parameters, the borrowing constraint  $\theta$  and the resaleability constraint  $\phi_t$ , jointly determine the amount of liquidity in the economy. We follow DEFK to set the steady-state values of  $\theta$  and  $\phi$  both to 0.185, which means that entrepreneurs can sell up to 56% ( $= 1 - 0.815^4$ ) of their equity holding in one year's time. Also following DEFK, a credit shock is modelled as a 60% drop in the value of  $\phi_t$  from 0.185 to 0.074 ( $e_t^\phi = -60\%$ ). In the DEFK model,  $\theta$  is fixed at its steady-state value even in a credit crisis. In this paper, we extend the analysis to study the effects of a tightening of borrowing constraints by lowering the starting value of  $\theta$  from 0.185 to 0.074.

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*Structural parameters:*

$\beta$	0.99	Discount factor
$\sigma$	1.39	Relative risk aversion
$\delta$	0.025	Depreciation rate
$\gamma$	0.36	Capital share
$\kappa$	1	Capital goods adjustment cost parameter
$\nu$	1.92	Inverse Frisch elasticity of labour supply
$\lambda_f$	0.11	Price mark-up
$\lambda_w$	0.11	Wage mark-up
$\zeta_p$	0.65	Price Calvo probability
$\zeta_w$	0.73	Wage Calvo probability

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*Parameters related to liquidity constraints:*

$\varkappa$	0.05	Probability of investment opportunity
$\theta$	0.185	Baseline borrowing constraint
$\phi$	0.185	Equity resaleability constraint at steady state
$L/4Y$	0.40	Steady-state government bonds to GDP ratio

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*Policy parameters:*

$\psi_\pi$	2.03	Taylor rule coefficient on inflation
$\psi_Y$	0.08	Taylor rule coefficient on output gap
$\psi_{\Delta Y}$	0.22	Taylor rule feedback coefficient on change in output gap
$\rho_R$	0.81	Interest rate smoothing
$G/Y$	0.18	Steady-state government spending share
$\rho_G$	0.97	Persistence of government spending shock
$\psi_k$	-0.063	Government open-market intervention coefficient
$\psi_\tau$	0.1	Government taxation rule coefficient

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Table 1: Calibration  
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Other parameters related to capital investment are  $\varkappa$ ,  $\kappa$ ,  $\gamma$  and  $\delta$ . Consistent with DEFK, we calibrate the i.i.d. opportunity to invest in each quarter ( $\varkappa$ ) to 0.05, which equals to a 20% opportunity to invest in a year. The capital adjustment cost parameter ( $\kappa$ ) is set to 1 as in DEFK. The capital share in the production function ( $\gamma$ ) and the quarterly depreciation rate ( $\delta$ ) takes on the conventional values of 0.36 and 0.025 respectively.

For the parameters that are standard in a DSGE model, we assign values mainly by referring to the mode of the posterior estimates obtained by SW. The coefficient of relative risk aversion ( $\sigma$ ) is 1.39, and the inverse Frisch elasticity of labour supply ( $\nu$ ) is 1.92. The Calvo probabilities for prices and wages ( $\zeta_p$  and  $\zeta_w$ ) are 0.65 and 0.73 respectively. Following Chari, Kehoe and McGrattan (2000), we assume the curvature parameters of the Dixit-Stiglitz aggregators in the goods and labour markets to be 10, meaning a markup of 0.11 in both goods and labour markets ( $\lambda_f = \lambda_w = 0.11$ ). We set the discount factor,  $\beta$ , equal to 0.99 as in DEFK.

We adopt the estimates of SW to the values of the parameters governing the conduction of monetary policy. The coefficients of inflation ( $\psi_\pi$ ) and output ( $\psi_Y$ ) in the monetary policy rule are 2.03 and 0.08 respectively; whereas the feedback coefficient on the change in the output gap ( $\psi_{\Delta Y}$ ) is 0.22. The degree of interest rate smoothing is estimated to be 0.81. Referring also to SW, the persistence of government spending shock is 0.97.

Since the government's quantitative easing policy is an invention of DEFK, we follow their calibration to set the coefficient on open market intervention ( $\psi_k$ ) to -0.063. As in the DEFK model, we assume that the taxation rule coefficient is low ( $\psi_\tau = 0.1$ ), which implies that the adjustment of taxes to the government's debt position is gradual.

The steady-state values of the endogenous variables are calculated based on the calibrated values of parameters. The results are reported in table 2. Two steady-state ratios are exogenous: the liquidity-to-GDP ratio ( $L/4Y$ ) and the government spending share in GDP ( $G/Y$ ). The liquidity-to-GDP

ratio shows the amount of government bonds issued as a share of the annual GDP in the steady state. Following DEFK, we set this ratio equal to 40%. The DEFK model does not consider government expenditure. So for the steady-state government spending share, we take the average value observed in the post-war United States which is around 18%.

Consumption to GDP ratio	$C/Y$	0.598
Investment to GDP ratio	$I/Y$	0.222
Quarterly GDP	$Y$	2.916
Quarterly employment	$H$	0.855
Capital stock	$K$	25.843
Total government bond issued	$L$	4.665
Tax to GDP ratio	$\tau/Y$	0.189
Real wage	$w$	1.965
Capital rent	$r^k$	3.656%
Cost of capital	$p^I$	1
Market price of equity	$q$	1.0699
Real marginal cost	$mc$	0.900
Real interest rate (annualised)	$r$	2.29%

Table 2: Steady-state values of endogenous variables in the DEFK model

A credit shock refers to a sudden worsening of the resaleability of equity, which is expressed by a drop of  $\phi_t$  to its low level (0.074). Evolution of  $\phi_t$  follows:

$$\widehat{\phi}_t = e_t^\phi < 0,$$

where  $\widehat{\phi}_t \equiv \frac{\phi_t - \phi}{\phi}$  is the percentage deviation of  $\phi_t$  from its steady-state value. Under a credit shock, households find it harder to locate a buyer for their

equity holdings and so they have less liquidity. Unlike DEFK, who assume  $\widehat{\phi}_t$  to follow a two-state Markov process, we assume that following a credit shock,  $\phi_t$  stays low for a deterministic number of periods depending on the expected duration of the crisis before it returns to its steady-state value.

A government spending shock occurs when the government unexpectedly increase or decrease its spending on goods and services. The deviation of government spending from steady state is measured as a percentage of GDP, denoted by  $\widehat{G}_t \equiv \frac{G_t - G}{Y}$ .  $\widehat{G}_t$  follows a first-order autoregressive process with a zero-mean i.i.d. error term  $e_t^G$ :

$$\widehat{G}_t = \rho_G \widehat{G}_{t-1} + e_t^G,$$

where the parameter  $\rho_G$  governs the persistence of government spending shocks. In our numerical experiments that follow, the size of government spending shock ( $e_t^G$ ) is set at 1% of GDP.<sup>3</sup>

In the original DEFK model, equilibrium conditions are log-linearised around the steady state. As shown later in our simulation experiments, the competitive equilibria achieved following a credit shock can stay far away from the steady state for a long time. Applying log-linear approximation in such a situation may lead to misleading results. For this reason, we do not follow DEFK to use log-linear approximations. Instead, we retain the nonlinear nature of the equilibrium conditions in our analysis.

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<sup>3</sup>Our chosen shock size is the same as Cogan et al. (2010) in the first part of their analysis. Christiano et al. (2011) and Woodford (2011) find, using log-linearised models, that the size of the government spending shock does not affect the multiplier as long as it does not change the duration of the zero-bound state. Erceg and Linde (2012) endogenise the duration of the liquidity trap and find that the multiplier is smaller when the size of the government spending shock increases. In view of this, we repeat the experiments by changing the size of the shock to 2% of GDP. Our results show that even the size of the shock has doubled, the drop in the multiplier is modest (only around 0.1).

### 3 How Large Is the Government Spending Multiplier?

How effective is expansionary fiscal policy in stimulating the economy? In our analysis, we measure the effectiveness of fiscal stimulus using the government spending multiplier. The fiscal multiplier is typically represented by the impact multiplier  $\frac{dY_t}{dG_t}$ , where  $dY_t \equiv Y_t - Y$  and  $dG_t \equiv G_t - G$  are the respective differences of output and government expenditure from steady state at a certain period  $t$ . As noted by Woodford (2011), this method of calculating the multiplier requires the time path of the increase in output to have the same shape as the time path of the increase in government spending in order to ensure that the multiplier has a clear meaning. We recognise that in most cases, the effects of fiscal stimulus are delayed, such that the time paths of the increase in output and the increase in government spending can differ substantially from each other. For this reason, we focus instead on the cumulative multiplier, which is defined as:

$$\frac{E_t \sum_{t=0}^{\infty} dY_t}{E_t \sum_{t=0}^{\infty} dG_t}.$$

Under this definition, the multiplier measures the expected cumulative increase in output given an expected one dollar cumulative increase in government expenditure. If the multiplier is greater than one, it implies that any change in government spending has a spillover effect; in other words, that a dollar increase produces a greater-than-one-dollar increase in GDP. In the following sections, we study the values of the fiscal multipliers under two scenarios: during normal times and at times of crisis when monetary policy becomes ineffective. We define normal times as the periods during which the economy is in the neighbourhood of the non-stochastic steady state. We

define credit crises as times at which the economy is hit by a credit shock (i.e., a 60% drop in the resaleability constraint parameter  $\phi_t$ ).

### 3.1 The Multiplier During Normal Times

In the DEFK model as well as in reality, credit frictions are present even during normal times due to the borrowing and resaleability constraints faced by households. We calculate the fiscal multiplier in normal times using the DEFK model by giving the steady state a positive government spending shock of 1% of GDP. Government expenditure shock follows an AR(1) process, as depicted in the previous section, and we assume that no subsequent shock is expected. The cumulative multiplier obtained using the DEFK model is 1.04.

How does our result compare with that obtained using a standard New Keynesian model that assumes no credit frictions? We carried out a control experiment by stripping all credit-constraint features from the DEFK model.<sup>4</sup> Given the same size of government spending shock, the model without financial frictions (henceforth the “standard” model) predicts a value of only 0.27 for the cumulative multiplier. This result supports the conclusions of the analytics performed by Woodford (2011), who observes that the fiscal multiplier will be less than one in a simple New Keynesian DSGE model in which monetary policy follows a standard Taylor rule.

Why is the multiplier obtained with a standard model so much smaller than the one we obtain from the DEFK model? Figure 1 shows the impulse-response functions (IRFs) of the key macroeconomic variables after a government spending shock for both the DEFK and the standard models. As seen from the IRFs, the predicted impacts of a government spending shock can differ greatly depending on whether or not financial frictions are present.

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<sup>4</sup>In this experiment, we used the same values of parameters as listed in table 1, with the exception of  $\beta$ , which was increased slightly to 0.9943 to keep the steady-state real interest rate in line with that in the DEFK model.

The IRFs in the standard model suggest that without financial frictions, both private investment and consumption expenditure are crowded out by fiscal expansion. This is because expansionary fiscal policy causes real interest rates to rise, thereby discouraging private investment and consumption. In addition, forward-looking households expect tax rises to follow in the future, leading to a negative wealth effect on private consumption. As a result, the increase in GDP is small and short-lived.

However, the IRFs generated by the DEFK model reveal the very different responses of some variables when credit frictions are added. Private investment furnishes a useful example. Private investment spending falls immediately after the shock, but rises in a hump-shaped manner after four quarters. At its peak, private investment is around 0.7% above its steady-state value and the positive effect is still observable as long as 25 quarters after the shock. Accordingly, the rise in GDP is much greater and more persistent. Immediately after the shock, the GDP gain predicted by the DEFK model is almost double that predicted by the standard model (0.4% vs 0.2%), and the difference in the projections seems to increase over time. This is why the fiscal multiplier obtained using the DEFK model (1.04) is around four times greater than that obtained with the standard model (0.27).

A closer look at the aggregate investment function (20) help us understand the crowding-in effects predicted by the DEFK model. Aggregate investment is a positive function of GDP and the real interest rate in the DEFK model. When the government increases its expenditure, it tends to increase the real interest rate along with the profits made by production firms due to an increase in overall production. Households therefore receive more interest income from their bond savings, as well as more dividend income from their share holdings in firms. The increased household income converts to extra liquidity for the credit-constrained entrepreneurs, since assets are distributed equally among household members at the beginning of each period. Since investment in new capital is more profitable than buying government bonds

or private equity, entrepreneurs invest all of their liquid assets in new capital, causing investment expenditure to rise. As a result, GDP increases further, giving a large fiscal multiplier in the DEFK model. In the standard model, however, physical capital is merely an alternative form of investment which gives the same rate of return as government bonds. An increase in the real interest rate following fiscal expansion increases in turn the opportunity cost of investing in physical capital. Under these conditions, private investment is therefore crowded out.

### 3.2 The Multiplier at Times of Crisis

We have shown that under normal conditions the government spending multiplier is higher with the presence of credit frictions. We now examine the value of this multiplier at times of credit crisis. To achieve this, we consider a 60% drop in the resaleability constraint parameter  $\phi_t$ . If the government decides to carry out fiscal stimulus measures in a credit crisis, we assume that the increase in government spending happens in the same quarter as the credit shock, i.e., at  $t = 1$ . The cumulative fiscal multiplier in a credit crisis is calculated by:

$$\frac{E_t \sum_{t=0}^{\infty} (dY_t - dY_t^*)}{E_t \sum_{t=0}^{\infty} dG_t}$$

$dY_t$  denotes the change in output from steady state due to the combined effects of the credit shock and the government spending shock, whereas  $dY_t^*$  denotes the same due to the credit shock alone by holding  $G_t$  constant. Therefore,  $(dY_t - dY_t^*)$  measures the output change in a credit crisis specifically due to the increase in government expenditure. The impact multiplier,  $\frac{dY_t - dY_t^*}{dG_t}$ , is calculated in a similar way by focusing on one particular period.

We simulate credit crises of various expected durations using the DEFK

model,<sup>5</sup> and compute both the impact multiplier and the cumulative multiplier by setting a government spending shock of 1% of GDP. Table 3 summarises the results. We note that the values of the multipliers increase consistently with an increase in the expected duration of the credit crisis. In the case in which the crisis is expected to last for just one year, the cumulative multiplier is 1.08 - a number not substantially different from that obtained in the normal-times case. With progressively longer-lasting credit crises expected, the value of the cumulative multiplier increases. For example, for a credit crisis lasting as long as the recent one, the value of the cumulative multiplier is around 1.5. If the expected duration of the crisis is seven years, the value of the multiplier is even higher, at around 2.

Expected duration of credit crisis	Expected duration of zero-interest rate	Post-shock impact multiplier	Cumulative multiplier
1q	1q	0.77	1.08
4q	1q	0.79	1.08
8q	3q	0.87	1.12
12q	8q	1.61	1.27
20q	16q	2.39	1.59
28q	24q	2.90	1.93

Table 3: Government spending multipliers under different expected durations of credit crisis

A similar pattern can be observed if we look at the post-shock impact multipliers. As the expected duration of the crisis increases, the impact multiplier show increases of a larger extent than the cumulative multiplier. If the credit crisis is expected to last for fewer than three years, the impact

<sup>5</sup>This experiment cannot be carried out using the standard New Keynesian model because this model does not allow for financial friction.

multiplier is smaller than one. If the crisis is expected to last for seven years, the impact multiplier will be around 3. Looking at the multipliers for the same expected crisis duration, we find that the impact multiplier is smaller than the cumulative multiplier whenever the crisis is expected to be shorter than three years. This implies that the multiplier effect of fiscal stimulus on output is delayed. Interestingly, the reverse is true if the crisis is expected to last for longer than three years. In such a case, the impact multiplier is larger than the cumulative one. This result suggests that the multiplier effect of fiscal stimulus is felt more quickly if the credit crisis is expected to last for a long time.

We also include in Table 3 the number of periods during which the zero lower bound on the nominal interest rate is binding in a credit crisis without fiscal stimulus. We find that the crucial factor affecting the value of the government spending multiplier is the expected duration of the zero-bound interest rate, rather than that of the crisis. This becomes clear when we compare the results for a one-quarter crisis to those for a one-year crisis. In both cases, the cumulative multiplier is equal to 1.08. This is possibly because the zero-bound condition lasts for only one quarter in both cases, despite the difference in the expected duration of crisis. Figure 2 plots the values of the multipliers against the expected duration of the zero-bound state. The positive sloping curves confirm the finding that fiscal policy is more powerful in a liquidity trap, especially if it is a long-lasting one.

To understand the reasons for the higher multiplier, we compute the IRFs of a credit crisis expected to last for three years. These values are plotted in Figure 3, which provides the IRFs under conditions both with and without fiscal stimulus<sup>6</sup>. As the figure shows, the nominal interest rate hits the zero-lower bound in response to a credit shock. Without fiscal stimulus, the nominal interest rate is zero-bound for eight quarters. An increase in govern-

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<sup>6</sup>Note that the IRFs are not smooth in this case. Most lines bend upwards at 12 quarters after the shock, when the economy is expected to exit from the credit crisis.

ment spending of 1% of GDP is able to help the economy exit the liquidity trap more quickly. The reason for a large fiscal multiplier in credit crisis conditions compared to normal times is also hinted at by the graphs. A rise in government spending raises output and expected inflation. Increased expected inflation reduces expected deflation, causing a drop in the real interest rate as the nominal interest rate is bound at zero. The reduced real interest rate stimulates private spending, with a knock-on effect on consumption expenditure, whose fall is reduced from 5.6% to 4.7% due to the fiscal stimulus. Consequently, the drop in GDP immediately after the credit shock reduces from 7% to 5.4%, giving an impact multiplier of 1.6.

Our discovery that the multiplier may be much larger at times of credit crisis is closely related to the findings reported in Christiano et al. (2011) and Woodford (2011). These studies find that the value of the fiscal multiplier is potentially much higher when the economy is stuck in a liquidity trap. However, our approach differs in that the cause of the liquidity trap arises from the conditions of model itself, while in Christiano et al. (2011) and Woodford (2011) it is exogenously determined. In our model, the multiplier values are smaller than those reported by Christiano et al. (2011) and Woodford (2011) since fiscal stimulus helps to push the economy out of the liquidity trap more quickly. An increased nominal interest rate would lead to an increase in the real interest rate, thereby reducing the effectiveness of fiscal policy as a stimulus to the economy. However, the multipliers in our model are still large enough for fiscal policy to be highly effective.

Another important point is that in order to obtain large multipliers, Christiano et al. (2011) and Woodford (2011) assume that fiscal expansion lasts exactly as long as the zero-bound state. The findings reported in these studies suggest that this assumption is crucial to large multiplier results. Indeed, Christiano et al. (2011) show that if the fiscal stimulus lasts longer than the liquidity trap, the fiscal multiplier decreases. Woodford (2011) suggests that Cogan et al. (2010) obtain small multipliers because

they assume that fiscal expansion continues for much longer than the zero-bound state. Cogan et al., assume that there is a permanent increase in government spending, whereas the zero lower bound is binding for 4 or 8 quarters only. As a result, the multiplier values they obtain are smaller. Our assumptions regarding government spending are in line with those of Cogan et al. (2010). We assume that government spending follows an AR(1) process. With a persistence parameter of 0.97, the increase in government spending is highly persistent and, in all of the cases we considered, lasts well beyond the duration of the liquidity trap. Thus, our finding suggests fiscal policy can be highly effective even when it continues for much longer than the period over which the interest rate is assumed to remain at zero.

### 3.2.1 The Multiplier Under Tightened Borrowing Constraints

So far in our analysis, we have kept the borrowing constraint parameter,  $\theta$ , constant at its steady state value even in times of crisis. Recall that  $\theta$  represents the maximum amount entrepreneurs can borrow in each period to fund their new investments. In reality, the difficulty of borrowing varies across economies as well as across industries. In light of these variation, we seek here to determine whether or not a change in  $\theta$  affects the value of the fiscal multiplier. We assume that  $\theta$  takes two values,  $\theta_H$  and  $\theta_L$ . In the benchmark case following DEFK, we set  $\theta$  equals to  $\theta_H = 0.185^7$ . The value of  $\theta_L$  is chosen to be equal to the resaleability constraint parameter  $\phi$  in times of crisis. As in the baseline case, we calculate the multipliers both at normal and in times of crisis.

The results of our simulation suggest that fiscal multipliers are generally larger fiscal multipliers in an economic environment in which borrowing is more difficult. Under these conditions, we find that, with  $\theta$  equal to  $\theta_L$ , the

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<sup>7</sup>Changing the value of  $\theta$  from 0.185 to 0.074, requires us also to change the capital share in the production function,  $\gamma$ , from 0.36 to 0.275, in order to keep the steady-state real interest rate the same as in the benchmark case (2.2%) for fair comparison.

cumulative multiplier at normal times is higher than in the benchmark case: 1.16 as opposed to 1.04. Similar results were obtained from the experiments simulating a credit crisis. We calculate the impact and cumulative multipliers in crises that last for one, three, five and seven years respectively and report the results in Table 4. In a crisis expected to last for three years, the cumulative multiplier increases from 1.27 in the benchmark case to 1.35 in the case with  $\theta_L$ . A similar increments size is observed when the expected duration of the crisis increases.

Expected duration of credit crisis	Expected duration of zero-interest rate	Post-shock impact multiplier	Cumulative multiplier
4q	1q	0.86	1.21
12q	6q	1.35	1.35
20q	15q	2.41	1.66
28q	24q	3.08	2.03

Table 4: Government spending multipliers in an economy with difficulty to borrow under different expected durations of credit crisis

It is important to note that in the most cases where  $\theta = \theta_L$ , compared with the baseline, we found the nominal interest rate to spend less time at its zero lower bound after the credit shock. This finding seems to suggest that the larger multiplier obtained using  $\theta_L$  is not due to the lengthening of the zero-bound state, but the tightening of borrowing constraints per se. One key implication of this result is that fiscal policy is more effective in economies facing tougher borrowing conditions.

## 4 Conclusions

In this paper, we have extended the DEFK model by introducing a role for government spending. We use the resulting model to study the effects of increasing government expenditure on the macroeconomy. The DEFK model accounts for credit constraints, generating a liquidity trap.

A number of interesting results arise from our analysis. First, the model employed in this paper suggests that fiscal policy is more effective than what the standard model with perfectly functioning financial markets suggests. At around 1, the fiscal multiplier obtained using the model employed in the paper is three times larger than that in the standard model. Second, the fiscal multiplier is much greater in a credit crisis, when the zero lower bound on the nominal interest rate is binding. Third, when conditions of credit crisis persist, the fiscal multiplier is sufficiently large for fiscal policy to be highly effective. Finally, we find that fiscal policy is even more effective in economies facing tougher borrowing conditions.

These findings strengthen the case for fiscal policy, especially in a prolonged credit crisis such as the current one.

## A Appendix

### A.1 Derivation of the Aggregate Investment Function

As investment in capital dominates saving in bonds or equity, a rational entrepreneur would use all the available resources to fund their investment in new capital. Entrepreneurs' net equity holdings thus evolve according to:

$$N_t(j) = (1 - \phi_t)(1 - \delta)N_{t-1}(j) + (1 - \theta)I_t(j),$$

where  $j \in [0, \varkappa)$ . It is also optimal for workers to buy all the consumption goods for the household so that entrepreneurs can spare more resources for capital investment. Hence, consumption expenditure is zero for entrepreneurs, i.e.,  $C_t(j) = 0$  for  $j \in [0, \varkappa)$ . Government bonds are not bound by resaleability constraint, so entrepreneurs sell all their bond holdings to fund their new investment. At the end of the period,  $L_t(j) = 0$  for  $j \in [0, \varkappa)$ . Putting these results into the intertemporal budget constraint (3), we obtain the aggregate investment function (4).

### A.2 Standard Features of the DEFK Model

#### A.2.1 Workers' wage-setting

Workers  $j \in [\varkappa, 1]$  supply differentiated labour  $H_t(j)$  to the production sector and receive nominal wages  $W_t(j)$ . Employment agencies combine  $H_t(j)$  into homogeneous units of labour input,  $H_t$ , using a constant elasticity of substitution (CES) aggregation function as proposed by Dixit and Stiglitz (1977). Employment agencies choose the profit-maximising amount of  $H_t(j)$  to hire, taking  $W_t(j)$  as given. The resultant demand for  $H_t(j)$  is decreasing with the relative wage of type- $j$  labour:

$$H_t(j) = \frac{1}{1 - \varkappa} \left[ \frac{W_t(j)}{W_t} \right]^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_t,$$

where  $-\frac{1+\lambda_\omega}{\lambda_\omega}$  measures the elasticity of substitution between different types of labour and  $\lambda_\omega \geq 0$ . Each type- $j$  labour is represented by a labour union who sets their nominal wage  $W_t(j)$  on a staggered basis. Each period, there is a history-independent probability of  $(1 - \zeta_\omega)$  for a union to reset their wage. Otherwise, they keep their wage constant. Let  $\widetilde{W}_t(j) \equiv P_t \widetilde{w}_t(j)$  be the optimal wage chosen by a labour union at period  $t$  that maximises their workers' utility, and  $w_t$  be the aggregate real wage. The first order condition, which is the same across labour unions, is as follows:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_\omega)^{s-t} C_s^{-\sigma} \left\{ \frac{\widetilde{w}_t}{\pi_{t,s}} - (1 + \lambda_\omega) \frac{\left[ \frac{1}{1-\varkappa} \left( \frac{\widetilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_s \right]^v}{C_s^{-\sigma}} \right\} \left( \frac{\widetilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_s = 0, \quad (11)$$

where

$$\pi_{t,s} = \begin{cases} 1, & \text{for } s = t \\ \pi_{t+1} \pi_{t+2} \dots \pi_s, & \text{for } s \geq t + 1 \end{cases}.$$

The zero-profit condition of employment agencies and the labour unions' wage-setting mechanism give rise to the dynamics of aggregate real wage:

$$w_t^{-\frac{1}{\lambda_\omega}} = (1 - \zeta_\omega) \widetilde{w}_t^{-\frac{1}{\lambda_\omega}} + \zeta_\omega \left( \frac{w_{t-1}}{\pi_t} \right)^{-\frac{1}{\lambda_\omega}} \quad (12)$$

### A.2.2 Firms' price-setting

Two groups of firms specialise in the production of intermediate goods and final goods respectively. Monopolistic competitive intermediate-goods firms hire labour and rent capital to produce heterogeneous goods  $Y_t(i)$  according

to the production function:  $Y_t(i) = A_t K_t(i)^\gamma H_t(i)^{1-\gamma}$ , where  $K_t(i)$  is capital input,  $H_t(i)$  is labour input,  $A_t$  is productivity and  $\gamma$  is the capital share. These firms maximise their real profits  $D_t(i)$  by choosing the optimal capital and labour inputs, and setting the optimal price for their specific goods. Capital and labour inputs are chosen to minimise the firms' production costs at given real wage ( $w_t$ ) and rental rate of capital ( $r_t^k$ ). The first-order conditions imply that  $\frac{K_t(i)}{H_t(i)} = \frac{\gamma}{(1-\gamma)} \frac{w_t}{r_t^k}$ . Accordingly, the marginal cost  $mc_t(i)$  is:

$$mc_t = mc_t(i) = \frac{1}{A_t} \left( \frac{w_t}{1-\gamma} \right)^{1-\gamma} \left( \frac{r_t^k}{\gamma} \right)^\gamma, \quad (13)$$

which is independent of the firm-specific production level. In each period, each firm can reset their price with a constant probability of  $(1 - \zeta_p)$ . Their price do not change otherwise. For the firms who reset their price, they choose the one that maximises their expected profits (weighted by the marginal utility of a representative household who owns the firms), considering that they might not be able to reset their price in the future. Define  $\tilde{p}_t(i) \equiv \frac{\tilde{P}_t(i)}{P_t}$  as the real optimal price chosen at  $t$ . The optimal price-setting condition is:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{p}_t}{\pi_{t,s}} - (1 + \lambda_f) mc_s \right\} \left( \frac{\tilde{p}_t}{\pi_{t,s}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_s = 0. \quad (14)$$

Final-goods firms produce homogeneous final goods,  $Y_t$ , by combining  $Y_t(i)$  according to the CES aggregation function and sell them at the market price,  $P_t$ . Their profit-maximising condition yields the demand function for intermediate goods:  $Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t$ , where  $-\frac{1+\lambda_f}{\lambda_f}$  measures the elasticity of substitution between different intermediate goods and  $\lambda_f \geq 0$ . The evolution of inflation is obtained from the zero-profit condition for final-goods producers:

$$1 = (1 - \zeta_p) \tilde{p}_t^{-\frac{1}{\lambda_f}} + \zeta_p \left( \frac{1}{\pi_t} \right)^{-\frac{1}{\lambda_f}} \quad (15)$$

### A.2.3 Production of capital-goods

Capital-goods producers convert final goods into capital goods. The adjustment cost of capital is quadratic in aggregate investment such that  $S(\frac{I_t}{I}) = \frac{\kappa}{2} (\frac{I_t}{I} - 1)^2$ , where  $I$  is the steady-state aggregate investment and  $\kappa$  is the adjustment cost parameter. Under this adjustment cost function,  $S(1) = S'(1) = 0$  and  $S''(1) > 0$ . These firms choose the amount of  $I_t$  to produce which maximises their profits  $D_t^K = [p_t^I - (1 + S(\frac{I_t}{I}))] I_t$ . The first order condition is:

$$p_t^I = 1 + S(\frac{I_t}{I}) + S'(\frac{I_t}{I}) \frac{I_t}{I} \quad (16)$$

## A.3 Aggregation and Resource Constraints

Capital evolves according to:

$$K_t = (1 - \delta) K_{t-1} + I_t \quad (17)$$

The market clears for both labour and capital so that  $H_t = \int_0^1 H_t(i) di$  and  $K_{t-1} = \int_0^1 K_t(i) di$ . The capital input required by firms at  $t$  is equal to the aggregate capital available at the *beginning* of  $t$  (i.e.,  $K_{t-1}$ ) because  $K_t$  is not yet determined until the end of the period.

The firms' optimal capital-labour ratio is independent of firm-specific variables, the aggregate capital-labour ratio is therefore simply:

$$\frac{K_{t-1}}{H_t} = \frac{\gamma}{(1 - \gamma)} \frac{w_t}{r_t^k}, \quad (18)$$

and the aggregate production function is:

$$A_t K_{t-1}^\gamma H_t^{1-\gamma} = \int_0^1 Y_t(i) di = Y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_f}{\lambda_f}} di. \quad (19)$$

The aggregate profits for intermediate-goods firms are wholly distributed to the households as dividends. Substituting for  $D_t$  and  $D_t^K$ , (4) becomes:

$$I_t = \mathcal{Z} \frac{[r_t^k + (1 - \delta) q_t \phi_t] N_{t-1} + \frac{R_{t-1} L_{t-1}}{\pi_t} + Y_t - w_t H_t - r_t^k K_{t-1} + p_t^I I_t - [1 + S(\frac{I_t}{I})] I_t - \tau_t}{p_t^I - \theta q_t} \quad (20)$$

Capital is owned either directly by the households, or indirectly by the government through their holdings of private equity:

$$K_t = N_t + N_t^g \quad (21)$$

The resource constraint requires that:

$$Y_t = C_t + \left[ 1 + S\left(\frac{I_t}{I}\right) \right] I_t + G_t. \quad (22)$$

Finally, the gross real interest rate is obtained by:

$$r_t = \frac{R_t}{E_t(\pi_{t+1})} \quad (23)$$

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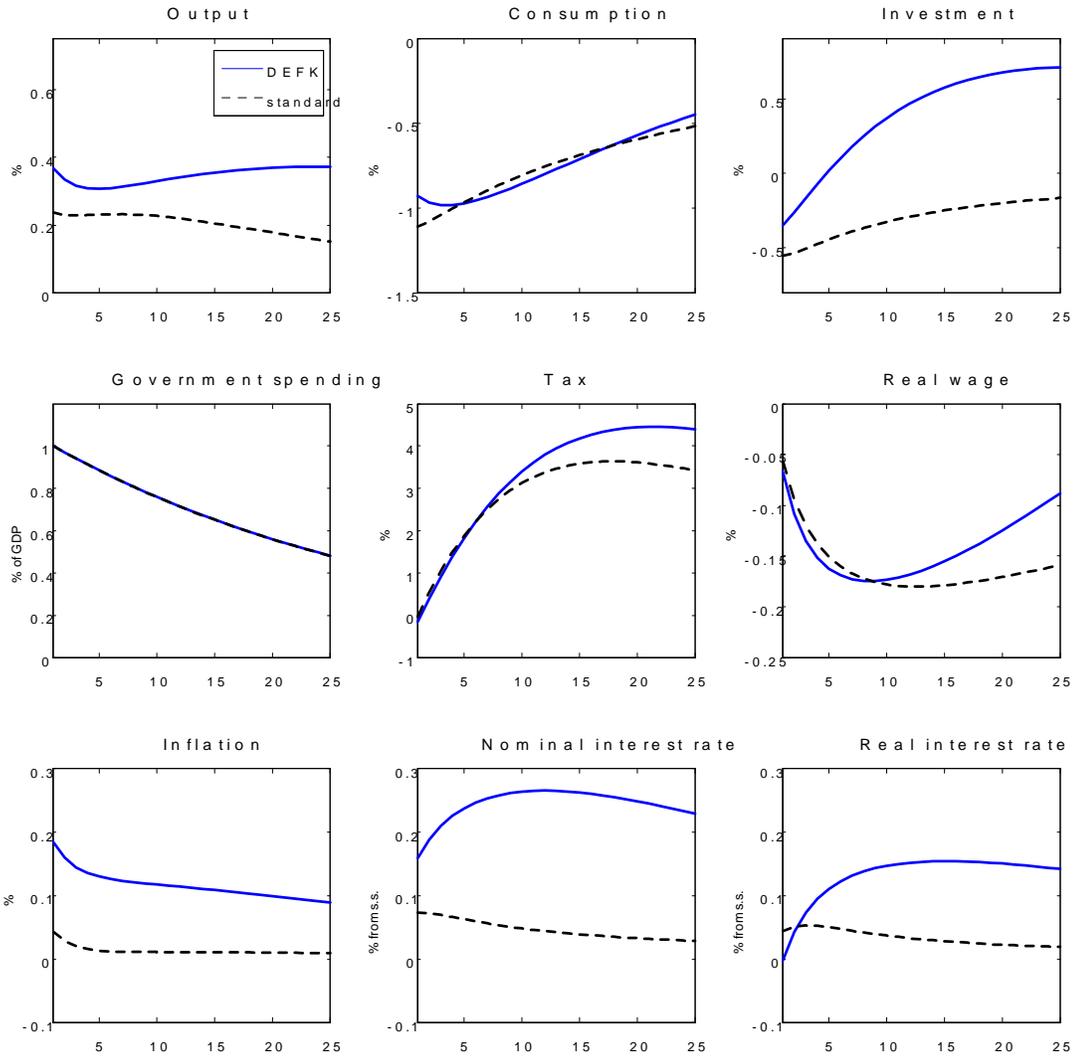


Figure 1: IRFs of a government spending shock in normal times: DEFK vs the standard model.

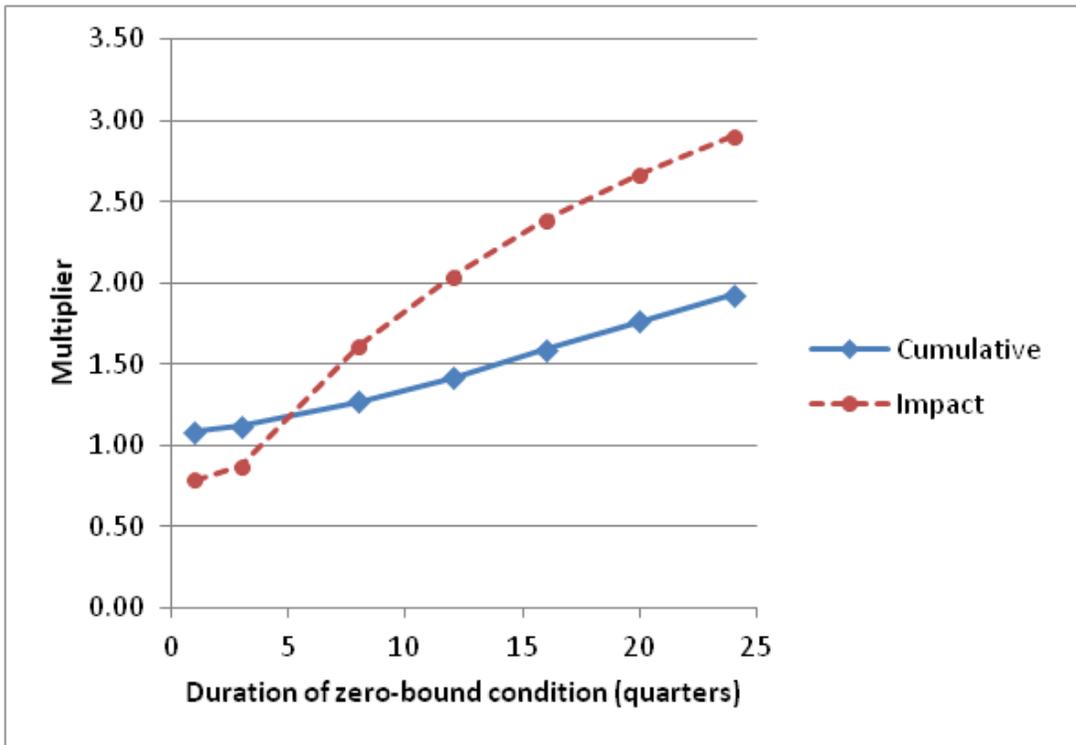


Figure 2: Fiscal multipliers and the expected duration of zero-bound condition

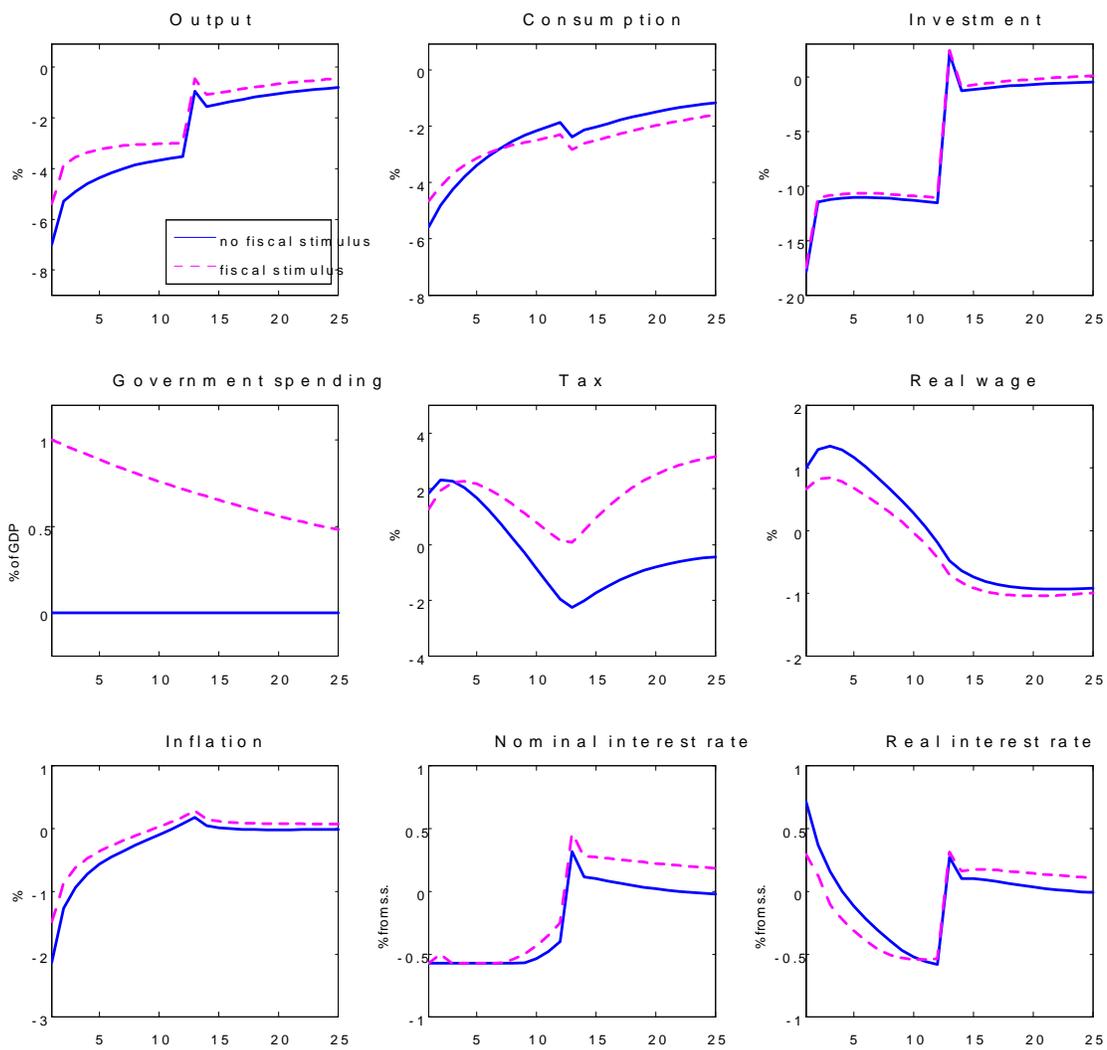


Figure 3: IRFs of a credit crisis expected to last for three years: the effects of fiscal stimulus.