Implementation Cycles in the New Economy+

Pasquale Scaramozzino
Department of Financial and Management Studies
SOAS, University of London, Thornhaugh Street, London WC1H 0XG, UK
and
Dipartimento di Economia e Istituzioni,
Università di Roma Tor Vergata, via Columbia 2, 00133 Roma, Italy
ps6@soas.ac.uk

Jonathan Temple“
Department of Economics, University of Bristol
8 Woodland Road, Bristol BS8 1TN, UK and CEPR
jon.temple@bristol.ac.uk

Nir Vulkan
The Said Business School and Worcester College
University of Oxford, Park End Street, Oxford OX1 1HP, UK
nir.vulkan@sbs.ox.ac.uk

26 September 2008

Abstract

The economic boom of the USA in the 1990s was remarkable in its duration, the sustained rise in equipment investment, the reduced volatility of productivity growth, and continued uncertainty about the trend growth rate. In this paper we link these phenomena using an extension of the classic model of implementation cycles due to Shleifer (1986). The key idea is that uncertainty about the trend growth rate can lead firms to bring forward the implementation of innovations, temporarily eliminating expectations-driven business cycles, because delay is risky when beliefs are not common knowledge.

JEL classifications: E32
Keywords: Implementation cycles, New Economy, multiple equilibria.

** We are grateful to Philippe Aghion for suggesting the use of Shleifer’s model to examine higher order beliefs and multiplicity, and to Francesco Giovannoni, Boyan Jovanovic, Alan Morrison, and especially David Myatt and Silvia Sonderegger for helpful contributions. The usual disclaimer applies. Temple also thanks the Leverhulme Trust for financial support under the Philip Leverhulme Prize Fellowship Scheme.

“ Corresponding author. Email jon.temple@bristol.ac.uk Telephone +44 117 928 8430
1. Introduction

The macroeconomic record of the USA in the 1990s was remarkable in a number of ways. The media attention given to the share prices of internet companies tended to obscure the achievements of the wider economy, including faster productivity growth, a rise in equipment investment, a reduction in output volatility, and an expansion that was sustained for exactly ten years – one of the longest on record. Although predictions of the ‘death’ or ‘taming’ of the business cycle were premature, there is strong evidence that the volatility of US output has been declining since at least the mid-1980s (McConnell and Perez-Quiros 2000). This period of unusual stability, the subject of much formal and informal commentary, has been dubbed the ‘Great Moderation’ (Stock and Watson 2002).

The New Economy period has directly inspired a growing number of papers which link business cycle fluctuations to the interaction between expectations and the implementation of new technologies. A particular focus has been the role of anticipated changes in productivity growth, and more general forms of “news”, in driving both output volatility and stock market fluctuations. Related papers include Beaudry, Collard and Portier (2006), Beaudry and Portier (2006), Francois and Lloyd-Ellis (2008), Jaimovich and Rebelo (2008), and Pástor and Veronesi (2008). The latter paper, in particular, is motivated by a particular feature of the New Economy period: despite the stability of output growth, there was uncertainty about whether this growth could be sustained, linked to uncertainty about the productivity of new technologies. As the duration of the boom exceeded all expectations, forecasters revised their predictions repeatedly, as we document further below. This implies the 1990s may have been an atypical period: unusually stable output growth was combined with a high degree of uncertainty about the trend growth rate.

In this paper, we use these observations to revisit a classic model of “intrinsic” business cycles due to Shleifer (1986). The starting point for Shleifer’s analysis is that firms must decide whether to implement innovations immediately, or wait for a
period of higher aggregate demand, when the profitability of implementation may be greater. As well as an equilibrium in which firms implement immediately, there can also exist multiple short-cycle equilibria, and sometimes also longer cycles. The cycle is entirely driven by expectations about the timing of a boom.

Recent work on expectations and multiple equilibria in macroeconomics has tended to emphasize the fragility of similar multiplicity results when agents are uncertain about the beliefs of other agents. With this in mind, we extend Shleifer’s model to incorporate the possibility of uncertainty about the underlying growth rate, motivated by the New Economy period. We will show that this uncertainty can eliminate cyclical equilibria, leaving immediate implementation as the only possible outcome. In Shleifer’s model, such an outcome would tend to be associated with a period of unusually stable, non-cyclical productivity growth, and a reduction in the volatility of investments associated with implementation. These were arguably features of the American boom of the 1990s; for example, Leduc and Sill (2007) attribute the reduction in output volatility in the US to a decline in the size of shocks to total factor productivity.

The remainder of the paper is structured as follows. Section 2 provides a more detailed review of stylized facts about the New Economy, helping to motivate our extension of Shleifer’s model. In section 3, we provide an overview of implementation cycles, emphasizing the role of expectations. Section 4 sets out the basic framework, before section 5 shows that uncertainty about the underlying growth rate leads to immediate implementation. Section 6 concludes.

2. Some stylized facts

In this section of the paper, we discuss evidence that is consistent with the model of business cycles due to Shleifer (1986), and that will inform and motivate our later theoretical analysis. We are especially interested in evidence that supports a central
result of Shleifer’s model. In his model, even when inventions arrive evenly over time, they are implemented in waves. The waves arise because firms have an incentive to defer implementation, if other firms are similarly deferring, until aggregate demand is relatively high.

We first ask whether there is evidence to support the view that new ideas are implemented with delays, and in waves. We review previous research, and also provide some new indirect evidence, by examining the behaviour of initial public offerings (IPOs) and productivity growth over the business cycle. We will argue that the cyclical patterns of these variables support the idea that innovations take place in waves.

More direct evidence on this point is hard to obtain. Survey-based counts of the successful commercialisation of inventions sometimes reveal a pattern of distinct peaks and troughs, as pointed out by Van Reenen (1996, p.219) using the data set for the UK described in Robson, Townsend and Pavitt (1988). This does not establish, however, that innovation clustering is the outcome of strategic delays.

In this respect, some interesting evidence is provided by the behaviour of stock markets in the wake of technological changes. Hobijn and Jovanovic (2001) explain major changes in US stock market valuations in terms of a delay between the creation of new technologies (such as information and communications technologies) and their implementation by new entrants. They argue that the potential of new technologies may be widely known several years before the technologies are implemented. This helps to explain the substantial decline in US stock valuations in the 1970s, given declines in expected profitability for incumbents and the market’s rational anticipation of entrants exploiting new technologies. This evidence is at least consistent with the view that implementation of new ideas involves delays, perhaps because entrepreneurs await favourable economic conditions, although other interpretations of the delays are also possible.
In exploring this idea in more detail, we focus mainly on US time series for movements in multifactor productivity (MFP) and initial public offerings (IPOs). We use both of these as proxies for the extent of innovative activity in the economy. We will be able to show that, especially after 1980, these two alternative measures tend to fluctuate in similar ways. Their co-movements support the idea that new technologies are implemented in waves. Furthermore, the extent of volatility in each series was lower in the 1990s than previously, consistent with our claim that clustering of innovations has diminished.

First of all, figure 1 plots MFP growth in the USA, for the private non-farm business sector, between 1960 and 2001. This shows the well-known tendency for marked year-to-year variation in MFP growth. This variation may reflect simply the random nature of technical progress. There could be sufficient randomness in the creation of new ideas that MFP growth varies substantially from year to year, even if implementation of a new idea is always immediate. An alternative view attributes the variation in MFP growth to measurement error of various kinds. Business cycles may be associated with systematic changes in measured MFP, notably through variation in factor utilization. There is evidence of this cyclical pattern in figure 1, at least before the mid-1980s. Measured MFP growth will then vary at short horizons even when underlying technical progress follows a smooth path and new ideas are implemented without delay.

---

1 The MFP growth series is constructed from historical Bureau of Labor Statistics data on MFP levels. The MFP data we use are based on the SIC classification, and are no longer updated by the BLS. We use them because they cover a longer span than the current measures based on the North American Industry Classification System (NAICS), which are not available before 1987. See the data appendix for more details and the data sources.

2 The main problem for this view is that it does not explain the significant positive autocorrelation seen in MFP growth, unless there are major technological shocks that have an economy-wide impact sustained over several years. As sometimes discussed in the real business cycle literature, it is not clear that innovations are sufficiently pervasive to generate the cyclical patterns seen in the aggregate data (see for example Stadler 1994).
Given these limitations of data on MFP growth, we combine this information with a more direct indicator of implementation, namely the number of initial public offerings (IPOs). Although IPOs vary in nature, a substantial fraction are clearly motivated by the desire to raise capital in the course of implementing a new business idea. Pástor and Veronesi (2005) note that around two-thirds of the leaders of IPOs cite the raising of capital as the main reason for an offering. Moreover, capital growth in the two years around the IPO is substantially higher than for comparable firms.

Figure 1 – Annual MFP growth rate, non-farm private business

Notes: This figure shows annual data on MFP growth for the non-farm private business sector, excluding government enterprises, calculated from historical BLS data. See Appendix 1 for more information on the data.

Figure 2 – Annual data on Initial Public Offerings
Notes: This figure shows annual data for the US economy on the number of IPOs, using data collected by Jay Ritter and made available on his website in 2007. See Appendix 1 for more details.

As with MFP growth, there is significant variation from year to year in the number of IPOs. Note that from the perspective of a finance textbook, the timing of an IPO should not matter, because any fairly-priced offering would not have a positive net present value. As Jenkinson and Ljungqvist (2001) note, this might suggest that the distribution of IPOs over time should be random. In fact, there is a well-known tendency for IPOs to cluster together in distinct waves. Both the year-to-year variation and the tendency for significant positive autocorrelation are apparent in figure 2, which plots annual data on the number of IPOs in the USA since 1960 (see Appendix 1 for the source of these data). At first glance, this supports a story in which entrepreneurs are willing to defer bringing an idea to the market.

Again, there are several possible explanations for the observed waves in IPOs. These include the possibility that entrepreneurs wish to take advantage of mispricing in equity markets. As Pástor and Veronesi (2005) argue, it is not clear why the
mispricing is clear to entrepreneurs but less readily observable to other market participants. Their preferred explanation is that the decision to go public can be seen as exercising a real option. Entrepreneurs might wish to delay an IPO, exercising the option only when there is a favourable change in market conditions. They present evidence that movements in expected aggregate profitability, including revisions to analysts’ earnings forecasts, are one determinant of the timing of IPOs. This endogeneity in the timing of investment can be seen as a specific instance of the general argument in Shleifer (1986).

Figure 3 – The co-movement of IPOs and MFP growth

Notes: This figure shows the co-movements of annual data on MFP growth for the private nonfarm business sector, and annual IPOs. These co-movements are stronger after 1980. See Appendix 1 for data sources.

If we see MFP growth and the number of IPOs as two alternative measures of the level of innovative activity in the economy, it is natural to ask whether the two move together in similar ways. We are not primarily interested in whether there is a causal relationship, but whether there is a tendency for these two series to move in the same way over time. If they move together, that will support our use of the two
measures as proxies for innovation. To the extent that each series indicates a ‘bunching’ of innovations, with similar timing, that will support our emphasis on cycles in implementation.

Figure 3 combines the annual data on IPOs with that on MFP growth. The correspondence between the two is weak for the 1960s and 1970s, but greatly strengthens thereafter, with a slight tendency for IPOs to anticipate movements in MFP growth. This relationship is stronger when we restrict attention to MFP growth in the manufacturing sector, disaggregated into durables and non-durables. Figures 4 (for non-durables) and 5 (for durables) again reveal the tendency for IPOs (for the whole economy) and MFP growth to move together after 1980.

Figure 4 – Annual MFP growth (manufacturing, non-durables) and IPOs

Notes: This figure shows the co-movements of annual data on MFP growth for the non-durables manufacturing sector, and annual IPOs. See Appendix 1 for data sources.

The visual impression is confirmed by two further ways of looking at the data. First, we report simple correlations between MFP growth and the (contemporaneous and
lagged) number of IPOs. Second, we will show that the number of IPOs helps to forecast MFP growth, even when conditioning on past MFP growth rates.

Figure 5 – Annual MFP growth (manufacturing, durables) and IPOs

Notes: This figure shows the co-movements of annual data on MFP growth for the durables manufacturing sector, and annual IPOs. See Appendix 1 for data sources.

Table 1 shows the correlations between MFP growth and lagged IPOs for the whole period (1963-2001) and for the subperiod 1980-2001. Given the likely measurement error in MFP growth, and the various influences on decisions to go public, the contemporaneous correlation for the post-1980 data is surprisingly high at 0.63. There is also some evidence that MFP growth is correlated with past numbers of IPOs, especially for the post-1980 period.
### Table 1 – Correlations between MFP growth and lagged IPOs

<table>
<thead>
<tr>
<th></th>
<th>1963-2001</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IPO(t-3)</td>
<td>IPO(t-2)</td>
<td>IPO(t-1)</td>
<td>IPO(t)</td>
<td></td>
</tr>
<tr>
<td>Business MFP(t)</td>
<td>0.14 (0.39)</td>
<td>-0.02 (0.89)</td>
<td>-0.00 (0.97)</td>
<td>0.10 (0.53)</td>
<td></td>
</tr>
<tr>
<td>Durables MPF(t)</td>
<td>0.30 (0.07)</td>
<td>0.24 (0.13)</td>
<td>0.45 (0.00)</td>
<td>0.40 (0.01)</td>
<td></td>
</tr>
<tr>
<td>Non-durables MFP(t)</td>
<td>-0.16 (0.34)</td>
<td>-0.08 (0.62)</td>
<td>0.14 (0.39)</td>
<td>0.24 (0.13)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1980-2001</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IPO(t-3)</td>
<td>IPO(t-2)</td>
<td>IPO(t-1)</td>
<td>IPO(t)</td>
<td></td>
</tr>
<tr>
<td>Business MFP(t)</td>
<td>0.37 (0.09)</td>
<td>0.29 (0.19)</td>
<td>0.33 (0.13)</td>
<td>0.63 (0.00)</td>
<td></td>
</tr>
<tr>
<td>Durables MPF(t)</td>
<td>0.28 (0.21)</td>
<td>0.28 (0.20)</td>
<td>0.71 (0.00)</td>
<td>0.46 (0.03)</td>
<td></td>
</tr>
<tr>
<td>Non-durables MFP(t)</td>
<td>-0.18 (0.42)</td>
<td>0.20 (0.37)</td>
<td>0.49 (0.02)</td>
<td>0.46 (0.03)</td>
<td></td>
</tr>
</tbody>
</table>

Notes. This table shows simple correlations between three MFP growth series (row) and IPOs in different periods (column) using annual data. Data sources are described in Appendix 1. Figures in parentheses are significance levels. Correlations significantly different from zero at the 10% level are shown in bold.

We now carry out simple Granger-causality tests, by regressing annual MFP growth on two lags of MFP growth and one lag of the number of IPOs. The intention here is not to make statements about causality in any structural sense, but to examine the extent of co-movement between the productivity growth and IPO series. To do this, we test the null hypothesis that the coefficient on lagged IPOs is equal to zero, using Newey-West standard errors to construct our test statistics. \(^3\) We also report the incremental $R^2$, the increase seen in the $R^2$ of the model for productivity growth by adding the lagged IPO variable. The results are shown in Table 2. For the period beginning in 1962, lagged IPOs help to forecast MFP growth only in the durables manufacturing sector (the zero restriction is not rejected in the other cases). For the

\(^3\) For these (Wald) test statistics to have their standard limiting distributions, the series must be stationary. For the various MFP growth series, we can easily reject the null of a unit root under a range of assumptions, using augmented Dickey-Fuller tests. For the IPO series, the results are slightly less clear-cut, but DF-GLS tests reject the null at the 10% level for a wide range of lag choices.
period after 1980, however, the IPO series helps to forecast all three MFP growth series (business, durables manufacturing, and non-durables manufacturing) and the effect on the R² of adding lagged IPOs is substantial.

Table 2 – Do IPOs help to predict future MFP growth?

<table>
<thead>
<tr>
<th>Regression</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector</td>
<td>Business</td>
<td>Business</td>
<td>Nondur</td>
<td>Durables</td>
<td>Business</td>
<td>Nondur</td>
<td>Durables</td>
</tr>
<tr>
<td>Observations</td>
<td>46</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>22</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Constant</td>
<td>0.01</td>
<td>(0.01)</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>MFPG(t-1)</td>
<td>0.16</td>
<td>0.17</td>
<td>0.40**</td>
<td>0.22</td>
<td>-0.27</td>
<td>0.20</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.15)</td>
<td>(0.11)</td>
<td>(0.12)</td>
<td>(0.21)</td>
<td>(0.21)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>MFPG(t-2)</td>
<td>-0.03</td>
<td>-0.06</td>
<td>-0.41</td>
<td>-0.22</td>
<td>0.10</td>
<td>-0.38</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.19)</td>
<td>(0.22)</td>
<td>(0.15)</td>
<td>(0.25)</td>
<td>(0.22)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>IPO(t-1)</td>
<td>-0.07</td>
<td>0.02</td>
<td>0.08</td>
<td>0.36*</td>
<td>0.33*</td>
<td>0.27*</td>
<td>0.58**</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.10)</td>
<td>(0.09)</td>
<td>(0.17)</td>
<td>(0.15)</td>
<td>(0.10)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>R²</td>
<td>0.03</td>
<td>0.03</td>
<td>0.24</td>
<td>0.27</td>
<td>0.16</td>
<td>0.37</td>
<td>0.53</td>
</tr>
<tr>
<td>LM(1)</td>
<td>0.81</td>
<td>0.85</td>
<td>0.43</td>
<td>0.49</td>
<td>0.66</td>
<td>0.70</td>
<td>0.24</td>
</tr>
<tr>
<td>LM(2)</td>
<td>0.53</td>
<td>0.53</td>
<td>0.69</td>
<td>0.45</td>
<td>0.50</td>
<td>0.26</td>
<td>0.46</td>
</tr>
<tr>
<td>Lagged IPO</td>
<td>0.44</td>
<td>0.84</td>
<td>0.36</td>
<td>0.04</td>
<td>0.05</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Incremental R²</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.13</td>
<td>0.16</td>
<td>0.22</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Notes. Dependent variable: MFP growth, MFPG. * significant at 5%; ** significant at 1%. Newey-West standard errors in parentheses, corrected for heteroscedasticity and serial correlation up to two lags. LM(n) is the p-value for a Breusch-Godfrey LM test for serial correlation, where the null hypothesis is no serial correlation of order n. “Lagged IPO” is the p-value for the null that IPO(t-1) has a zero coefficient, based on Newey-West standard errors. “Incremental R²” is the increase in the R² achieved by including IPO(t-1) in the model. In presenting the results, the IPO series has been rescaled by dividing by 10000.

This result is not necessarily surprising, given that IPOs are inherently forward-looking, and we are not claiming to have identified a genuine causal effect. We are interested in these correlations for the more general relationship that is revealed: the extent to which IPOs and MFP growth fluctuate in similar ways over the business cycle. The tendency for these series to move jointly, with periods of high IPO activity preceding high MFP growth rates, supports the idea of innovative activity that is ‘bunched’ in distinct periods or waves. The evidence we present suggests that bunching of innovative activity is initially reflected in the observed timing of IPOs and subsequently in MFP growth.
We now turn to a further set of stylized facts, related to a central argument of our paper. We will argue that implementation cycles were weakened in the 1990s, consistent with a tendency for innovations to be implemented rapidly rather than deferred to better times. This shows how Shleifer’s model might be used to interpret the stylized facts of the New Economy period. Although a direct test of this hypothesis is hard to implement, we can at least examine whether the aggregate data are consistent with weaker implementation cycles.

It is well known that the 1990s were a period of unusual stability for the US economy, the ‘Great Moderation’ that is documented in McConnell and Perez-Quiros (2000), Blanchard and Simon (2001) and Stock and Watson (2002), among others. To the extent that implementation has been smoother and less subject to distinct waves, we would expect to see reduced volatility in our proxies for implementation, namely MFP growth and IPOs. We now examine whether volatility in these measures declined over the course of the 1990s, a necessary condition for claiming that implementation cycles have weakened.

We will use as our measure of volatility the (scaled) median absolute deviation:

\[ 1.484 \cdot \text{median}|x_i - \text{median}(x)| \]

or in other words, the median of the absolute deviations from the median. This is a standard robust estimator of the scale of a distribution, which is less influenced by any single observation than the standard deviation. The scaling factor 1.484 ensures

---

4 Direct evidence is hard to find. For example, Jorgenson and Stiroh (2000, p. 158) note a shortening in the product cycle of microprocessors, with new processors brought to market more quickly in the 1990s than previously; but this may reflect an acceleration in technical change rather than an elimination of implementation lags.

5 Note that the hypothesis of declining volatility is conceptually separate from our earlier hypothesis that IPOs and MFP growth move together. It is possible for the covariance of these two series to increase (as appears to have happened in 1980-2001, compared to the earlier twenty-year period) at the same time as the short-run volatility of each series shows a decline.
that the statistic will be a consistent estimator of the standard deviation of a normally distributed variable.\textsuperscript{6}

First of all, figure 6 plots a 9-year rolling median absolute deviation of MFP growth in the private non-farm business sector. An especially noteworthy aspect of this figure is that MFP growth was stable, by historical standards, even through the recession of 2001. This supports our argument that implementation cycles were temporarily in abeyance.

Figure 7 restricts attention to the volatility of MFP growth in the manufacturing sector, disaggregated into durables and non-durables, again using a 9-year rolling median absolute deviation for each series. Figure 8 examines the volatility of the annual number of IPOs. All the figures reveal the same pattern, namely that volatility was noticeably lower in the 1990s than previously, although the volatility of IPOs shows an increase as the boom finally draws to a close.

\textsuperscript{6} We use the median absolute deviation because of the small number of observations and the possibility that the year-to-year variation in the series, such as MFP growth rates, might contain significant measurement errors. The patterns we describe below are also apparent when we use the standard deviation as a measure of volatility (see the working paper version of this research).
These findings are consistent with other research; for example, Leduc and Sill (2007) present evidence that reduced output volatility in the US should be attributed to smaller shocks to total factor productivity. In the remainder of the paper, we will present a theoretical argument that could explain this reduced volatility. The argument relies on uncertainty over the underlying rate of productivity growth, which can eliminate the multiplicity of equilibria obtained by Shleifer. At first sight, our argument might appear to be on unsafe ground, because superficially the 1990s were a period of stability rather than uncertainty. Here, however, the distinction between volatility and predictability is crucial. It is well known that a series can be volatile but predictable, but in the 1990s the reverse obtained. The New Economy period was one in which major macroeconomic variables were unusually stable, to an extent that caught out many observers. As stated by Robert Hall in his comments on Blanchard and Simon (2001), five-year and ten-year forecast errors for the US economy were unusually large in the 1990s.
Much the same point is made in Jorgenson and Stiroh (2000, p. 162-165). They note that forecasters repeatedly had to raise growth projections, and that the Congressional Budget Office revised forecasts of TFP growth upwards on a number of occasions. The uncertainty arose partly because the 1990s expansion was sustained to an unusual extent, making it harder to rely on past cycles as a guide. Moreover, since growth consistently exceeded expectations, there was speculation that trend growth had increased, and disagreement over the extent to which this had happened.

The combination of a sustained expansion, and a massive stock market boom, led to wide discussion of the possibility that trend growth had increased, in both the business press and more academic commentary. Views differed, indicating the uncertainty even among close observers. In reviewing productivity growth in the 1990s, Jorgenson and Stiroh (2000) argued that there was a case for an upwards revision of medium-term growth forecasts. In contrast, Gordon, in his comments on the same paper, argued that some of the productivity gains of 1995-99 were likely to prove transient, and that the reputation of the New Economy had been inflated by cyclical factors. More recently, it is clear from figure 1 that productivity growth continued its strong performance despite the 2001 recession, another departure from previous cyclical patterns.
Figure 7 – The declining volatility of MFP growth in manufacturing

Notes: The plotted values at date T are the nine-year rolling median absolute deviation of the two MFP growth series using data from year T-8 to year T. See Appendix 1 for data sources.
Figure 8 – The declining volatility of IPOs

Notes: The plotted value at date T is the nine-year median absolute deviation of the annual IPO series using data from year T-8 to year T. By the year 2000, volatility on this measure is clearly much lower than ten years previously, although volatility increases again as the boom draws to a close.

As noted by Sichel in his commentary on Jorgenson and Stiroh (2000), the decomposition of output growth into trend and cyclical effects is particularly difficult when the length and nature of an expansion has departed so sharply from previous norms. Stiroh (1999), in discussing the possibility of a rise in trend growth, argued that conclusions would have to await new evidence. Combined, the lack of consensus illustrates the uncertainty about the trend growth rate that was an important feature of the late 1990s.

The theoretical analysis in the remainder of the paper will explain why uncertainty of this kind could have implications for Shleifer’s explanation of business cycles. Although it may seem paradoxical at first sight, the Shleifer model can explain the unusual stability of the 1990s, if we appeal to contemporaneous uncertainty about
the underlying trend growth rate; this paradox arises because the uncertainty about trend growth can potentially eliminate cyclical equilibria.

3. Implementation cycles

In this section, we provide an overview of the arguments in the remainder of the paper. The arguments build on a long tradition in macroeconomics, emphasizing the importance of expectations and beliefs for macroeconomic behaviour. This has been stressed at least since Keynes (1936) argued that "animal spirits" may give rise to instability. Expectations of booms and recessions can be self-fulfilling, as agents bring forward or postpone their investment decisions, depending on their perceptions of how the economy will evolve in the future. If some firms anticipate an increase in aggregate demand, they may decide not to invest in the present period and delay their investment to some future date. This will enable those firms to maximize the revenue from their sales during a boom. If other firms in the economy share the same expectations about future demand, they will also postpone their investment to the future. This will bring about a recession in the current period and a boom at a later date.

Based on this kind of intuition, there is now a large literature on self-fulfilling prophecies, stemming from the theoretical analyses of Azariadis (1981) and Cass and Shell (1983), and surveyed by Farmer (1993), Silvestre (1993) and Matsuyama (1995). Many of these models imply that, under some conditions, there are several possible outcomes or even a continuum of equilibria.

The practical relevance of multiplicity has been questioned by examining the role of higher order beliefs (beliefs about beliefs). Recent contributions emphasize that certain equilibria will be observed only under restrictive assumptions on the
informational structure of the economy. Coordination on certain equilibria often requires an assumption that agents have common knowledge about the fundamentals of the economy and about the beliefs (of all orders) of the other agents. In particular, the expectations of all the agents in the economy should be common knowledge, in the technical sense of that term.

This is clearly an unrealistic assumption to make in macroeconomic models. A more satisfactory assumption is that agents have imperfect knowledge of the fundamentals of the economy and of the beliefs held by everybody else. Their beliefs may still be related to those of other agents: individuals can learn about the information and beliefs of others, simply by observing their actions. Furthermore, they share access to public information. The key point, however, is that the beliefs of all agents are unlikely to be common knowledge.

This apparently minor change in assumptions has dramatic implications. Imagine that agents receive noisy signals about the same key parameter, and the noise affecting the signal is idiosyncratic so that agents’ signals may be different. In this case, and under quite general conditions, agents will select what they perceive to be their least risky course of action. As a consequence, some of the equilibria in the economy can be ruled out.

In the analysis that follows, we apply these ideas to the multiplicity of equilibria in Shleifer’s model of implementation cycles. His framework is particularly appropriate for looking at the role of information assumptions in macroeconomics, since the cyclical equilibria rely on expectations about expectations.

---

7 The fragility of some equilibria in the presence of uncertainty and correlated signals has been analyzed by several authors in different contexts. Shin (1995) considers a decentralized economy with search externalities. Morris and Shin (1998) look at the timing of speculative attacks against a currency. Scaramozzino and Vulkan (2004) examine a model of local oligopoly with correlated noise about the competitive advantage of firms. See Morris and Shin (2000, 2003) for details of more applications, especially to macroeconomic issues.
In Shleifer's model, the rate of technological progress is a known constant. In the analysis that follows we demonstrate that, if there is uncertainty about the rate of technological progress, and if signals about this variable are correlated across agents, then agents will coordinate on a single equilibrium. Under some conditions, immediate implementation is the only undominated strategy for firms. According to this result, it would not be profitable for firms to delay the implementation of their innovations. The potential relevance to the New Economy period should be clear. The uncertainty about the trend growth rate, by encouraging firms to implement immediately rather than delay, could eliminate implementation cycles and be associated with an unusually long expansion.

The intuition for our results can be summarized as follows. Suppose that we are in a situation where the fundamentals of the economy are only consistent with immediate implementation, and this is the dominant strategy for firms. Suppose now that the fundamentals change slightly, and that immediate implementation is only "almost" dominant. Firms might choose to delay the implementation of their innovations. Yet, if there is some noise about the fundamentals, and if agents are uncertain regarding the beliefs of the other agents in the economy, delaying the implementation is a riskier strategy than immediate implementation. Firms will therefore tend to implement immediately.

More generally, the optimal strategy depends on what other firms will do in nearby states of the world, including those in which immediate implementation is a dominant strategy. Taking these into account, the \textit{ex ante} dominant strategy is not to wait for a boom. This logic applies even to circumstances in which the fundamentals of the economy are not close to making immediate implementation "almost" dominant, as we clarify below.

4. The basic setup
The basic structure of the model is identical to Shleifer (1986), and we refer the reader to that paper for full details. Briefly, an infinitely-lived representative consumer maximizes utility:

\[
\sum_{t=1}^{\infty} \rho^{t-1} \left( \prod_{j=1}^{N} x_{tj}^\gamma \right)^{1-\gamma} \frac{1}{1-\gamma}
\]

where \(0<\rho<1\) is the subjective discount factor, \(0 \leq \gamma < 1\) indexes the extent of relative risk aversion, \(x_{tj}\) is the consumption of good \(j\) in period \(t\), \(N\) is the number of commodities, and \(\lambda \equiv 1/N\), where \(N\) is a large number. Preferences are assumed to be Cobb-Douglas to ensure that equilibrium in each sector is determined by aggregate demand. There are perfect capital markets. The lifetime budget constraint of the representative agent is:

\[
\sum_{t=1}^{\infty} \frac{y_t - \sum_{j=1}^{N} p_{tj} x_{tj}}{D_{t-1}} = 0
\]

where \(p_{tj}\) is the price of commodity \(j\) in period \(t\), \(y_t\) is income, and \(D_t = (1+r_t)\ldots(1+r_1)\) is the inverse of the discount factor, where \(1+r_t\) is the rate of interest paid in period \(t+1\) and where \(D_0\) is set equal to unity. Consumption at time \(t\) is given by \(c_t = \sum_{j=1}^{N} p_{tj} x_{tj}\).

The structure of preferences implies constant expenditure shares:

\[
p_{tj} x_{tj} = \lambda c_t
\]

No storage technology is assumed to exist: hence, \(c_t = y_t\) and the consumer is neither a borrower nor a saver. As in Shleifer (1986), the equilibrium interest rate is thus:
(4) \[ 1 + r_t = \frac{1}{\rho} \cdot \left( \frac{y_{t+1}}{y_t} \right)^\gamma \cdot \left( \frac{\prod_{j=1}^N p_{j,t+1}}{\prod_{j=1}^N \prod_{j=1}^N p_{j,t}} \right)^{1-\gamma} \]

Let \( \Pi \) be aggregate profits. Labor is inelastically supplied at \( L \). A unit of labor is the numeraire and so the wage rate is normalized to unity. The income identity is then given by:

(5) \[ y_t = \Pi_t + L \]

There are \( N \) ordered sectors in the economy. In the first period, one firm in each of the sectors 1, 2, ..., \( n \) generates an invention (so there are \( n \) inventions in the first period). In the second period, one firm in each of the sectors \( n+1, n+2, ..., 2n \) generates an invention. In period \( T^* = N/n \) one firm in each of the sectors \( (T^* - 1)n + 1, (T^* - 1)n + 2, \ldots, T^* n \) generates an invention, before the cycle repeats starting in period \( T^* + 1 \). An invention in period \( t \) enables firms to produce output using a fraction \( 1/\mu \) of the labor input which was previously required, where \( \mu > 1 \) is the rate of technical progress. It is this rate that we will model as uncertain in the analysis of the next section.

Firms that invent can implement immediately or delay. When a firm implements its invention, it becomes a monopolistic supplier in its sector. Its profits are

(6) \[ \pi_t = m \cdot y_t \]

where \( m = \lambda(1 - 1/\mu) \). In the period following the implementation, imitators enter the market and drive the profits of the innovating firm down to zero. Hence, firms have an incentive to maximize the short-run returns from implementing the innovation. They will trade off the opportunity cost of delaying the innovation to the
future against the potential gain from implementing during a period of high aggregate demand.

Let \(0 \leq \alpha \leq 1\) be the fraction of the \(n\) firms receiving an invention at time \(t = 1, \ldots, T-1\) that implement immediately. Let \(\beta_T = \left( T - \sum_{t=1}^{T-1} \alpha_t \right) \) so that \(\beta_T \cdot n\) denotes the number of firms that implement at time \(T\): those who received an invention during the cycle and waited, and those who received an invention at time \(T\). Note that \(\beta_T = 1\) when all firms implement immediately and \(\beta_T = T\) when they all wait until time \(T\).

Cycles of period \(T \leq T^*\) are an equilibrium if and only if \(\pi_T / D_{T-1} > \pi_1\), or

\[
\rho^{T-1} \left( \frac{1 - nm\beta_T}{1 - nm\alpha_i} \right)^{\gamma - 1} \mu \left( 1 - n \right) \sum_{i=1}^{\gamma} \frac{\alpha_i}{\lambda} > 1
\]

Equation (7) simply requires that, prior to the boom, interest rates are not high enough to offset the incentive for firms to wait until the boom to receive their profits (Shleifer, 1986, p. 1172). By investigating the left-hand-side of equation (7) we can make a number of useful observations about the degree of coordination required to sustain a \(T\)-boom equilibrium.

First, if everyone waits until time \(T\), i.e. \(\alpha_1 = \alpha_2 = \ldots = \alpha_{T-1} = 0\) and \(\beta_T = T\), then equation (7) collapses to Shleifer’s equation (12): \(f(T) \equiv \rho^{T-1} (1 - nTm)^{\gamma - 1} > 1\). Second, if no one waits or \(\alpha_1 = \alpha_2 = \ldots = \alpha_{T-1} = \beta_T = 1\) then the LHS < 1.
Since the LHS is continuous in $\sum_{i=1}^{T-1} \alpha_i$, then there exists a $0 \leq k \leq 1$ such that LHS$<1$ if and only if $k \leq \frac{\sum_{i=1}^{T-1} \alpha_i}{T-1}$. In other words, a $T$-boom can be supported as a Nash equilibrium if and only if at least a fraction $k$ of the $n(T-1)$ firms receiving an innovation at periods $1, \ldots, T-1$ wait. The precise value of $k$ will depend on the parameters of the model, and from now on we will restrict attention to the case where $k$ is greater than 0.5.\(^8\)

On the other hand immediate implementation ("cycles" of length 1) always form an equilibrium in Shleifer's model. The payoffs for a firm that chooses to implement immediately do not depend on the behaviour of any of the firms receiving an innovation in the same period or in the future.\(^9\)

An important point is that cycles of size $T\geq2$ require a much greater degree of coordination than an equilibrium with immediate implementation. It is this need for coordination that makes the cyclical equilibria potentially fragile. In the next section, we will argue that when there is uncertainty about the rate of technical progress, only the equilibrium where firms implement immediately is robust.

5. **Extending the basic model**

This section reconsiders Shleifer's cyclical equilibria, from the viewpoint of the literature on global games. Our departure from Shleifer (1986) is that we assume the rate of technical progress $\mu$ is not known to firms. Instead, they receive a noisy signal about $\mu$. Given equation (6), this corresponds to a noisy signal on profits from implementation, and our discussion will proceed in terms of the parameter $m$.

---

\(^8\) This is not unduly restrictive. Numerical simulations suggest that $k$ is substantially higher, around 0.75 for many parameter values that satisfy Shleifer's parameter restrictions.

\(^9\) Of course, the payoff from implementing immediately will be higher if firms that received innovations in the past waited until this period.
Formally, we assume that Nature draws $m$ once at the beginning of each cycle of inventions – that is at time 1, $T^*+1$, $2T^*+1$ and so on - according to a uniform distribution over $[m,\bar{m}]$. A firm that receives an innovation, also receives a signal on $m$ and then chooses whether or not to delay implementing. Let $M$ be a one-dimensional random variable and let $(E_i)_{i=1}^{n(T-1)}$ be an $n(T-1)$-tuple of i.i.d. random variables, each having zero mean. Each $E_i$ is independent of $M$, with a continuous density and a support within $[-1, 1]$. For $\varepsilon > 0$ we write:

$$M'_i = M + \varepsilon E_i$$

If $\varepsilon = 0$ then $m$ is common knowledge and we are back to Shleifer’s model. We are interested in what happens when $\varepsilon$ is arbitrarily small – that is, under almost common knowledge.

Denote by $\Omega$ the set of all values of $m$ for which $f(T)<1$ for $T=2, \ldots, T^*$. Let $m^*$ be the infimum of the set $\Omega$. We shall now assume that $m^* \in (m, \bar{m})$. Notice that for $m < m^*$, immediate implementation is the dominant strategy in Shleifer’s model: $f(T)<1$ for all $T \geq 2$ means that $\pi_T / D_{T-1} < \pi_1$, or that implementing immediately yields a higher payoff than waiting until period $T$ even if everyone else is waiting. We have seen from equation (7) that if some firms wait, the payoff from waiting will increase and the payoff from implementing immediately either does not change or decreases. But if $m < m^*$ the strategy “implement immediately” yields a higher payoff than the strategy “wait until period $T$” for all $T$, regardless of the behaviour of the other firms – in other words, it is the dominant strategy.

---

10 In fact, in many cases it is sufficient that $f(2)<1$ for this to hold. See Figure 2, page 1176 in Shleifer and the discussion found there.
In our extended model this is still true: if a firm $i$ receives a signal $m_i < m^*$ then it expects that the true value of $m$ is smaller than $m^*$, because $E(M|m_i = m_i) = m_i$. The firm will therefore implement immediately.

In Shleifer’s model, for $m \geq m^*$ longer cycles, or $T$-booms ($T = 2, 3, \ldots, T^*$) can also be sustained as a Nash Equilibrium. As we now show, this is no longer the case in our extension, when there is noise.

In deriving this result, it is important to note that we make a crucial, but artificial, simplifying assumption. We shall assume that firms select their strategies (that is, mappings from signals to behaviour) at the beginning of each cycle. This can be seen as consistent with the spirit of Shleifer’s original model, but we rule out the more realistic case in which firms make their choices after receiving their signal, and in particular, after observing the actions of firms receiving innovations in previous periods. The more realistic case would require a more complicated proof: for example, a firm which receives an innovation in a given period will know that if it waits, this will affect the behaviour of firms receiving innovations in later periods.

Under more general assumptions, the arguments we use below, and in particular the symmetry argument we use to justify equation (10), would have to be modified to take into account the posterior distributions of firms that receive innovations later in the $T$-boom. This would correspond to applying the global games arguments to a more complex case where groups of players move sequentially, rather than all moving simultaneously.

It is possible that the general line of argument can be applied in such a case, as in Chamley (2005), but any extension to sequential moves will complicate the analysis. Some researchers have extended global game results to repeated games, but in the context of problems that are in some respects simpler. References include Giannitsarou and Toxvaerd (2007) and the papers cited there. These extensions often
consider a continuum of agents making decisions in what is essentially a repeated game (plus noise). In this paper, we do not assume a continuum, or that the game is the same in each period, and the proof in our paper relies heavily on the fact that there is a finite number of participating agents at each stage. The alternative assumption of a continuum of agents does simplify the analysis in some respects, but also misses out on the type of reasoning used in this paper: that some actions are more risky based on what I think the others think of me, which depends on what I think of them, and so on.

Extending this type of reasoning to the full dynamic case, with updating of beliefs, is not straightforward and has not yet been resolved. We therefore focus on the case in which firms choose their strategies in advance of receiving their signal, and do not take into account observed actions of other firms. In terms of eliminating cycles, this may or may not be a conservative assumption: arguably, a firm that observes another firm implementing immediately is less likely to delay, but at the same time, a firm choosing to delay knows that this could encourage firms in future periods to delay. By ruling out these considerations, our assumption about the timing of strategy choices will keep the logic closer to that in the existing global games literature.

We can now prove our main result:

**Proposition.** In the implementation cycle model with noisy signals, the only possible equilibrium is one with immediate implementation.

**Proof.** By contradiction: Assume that there exists $\tilde{m} \geq m^*$ and a symmetric Nash equilibrium $S$ where any firm $i$ receiving an invention at time $t$ ($\neq T-1$) and a signal $m_i \geq \tilde{m}$ delays its implementation until time $T>1$. 

Denote by $\phi_i(m_i, s_{-i})$ the probability firm $i$ attaches to the event that more than $kn(T-1)$ firms that receive innovations at periods $1, \ldots, T-1$ wait for a $T$-boom, when its own signal is $m_i$ and their equilibrium strategies are $s_{-i}$.

**Lemma.** $\phi_i(m_i, s_{-i}) > 0$

**Proof.** Given the structure of Shleifer’s model, this result is a corollary of the maintained assumption that $S$ is an equilibrium. A more formal proof is provided in Appendix 2.

Suppose now that $m_i - m^* < 2\varepsilon$.

\begin{equation}
\phi_i(m_i, S_{-i}) \leq Pr(\text{at least } kn \cdot (T-1) \text{ receive a signal } > m_i \mid M_i = m^*)
\end{equation}

\begin{equation}
= Pr(E_i > E_i \text{ for at least } kn \cdot (T-1) \text{ components of the vector } E)
\end{equation}

\begin{equation}
= \sum_{m=kn}^{n} \binom{n}{m} \frac{1}{2^n}
\end{equation}

Equation (9) follows from the fact that firms which receive a signal $m_i < \bar{m}$ implement immediately (because it is a dominant strategy). As long as $m_i$ lies inside the support of $M$, this probability is independent of the exact value of $m_i$.

The expression in (10) follows from the symmetry of $E$.

Finally we note that (11) $\sum_{m=kn}^{n} \binom{n}{m} \frac{1}{2^n}$ converges to zero as $n$ increases for a fixed $k > 0.5$. Formally, using the central limit theorem, this sum converges to:

$$\frac{1}{\sqrt{2\pi}} \int_{(2k-1)\sqrt{u}}^{\infty} e^{-\frac{1}{2}t^2} dt$$

which essentially is a step function equal to 1 for $k < 0.5$ and 0 for $k > 0.5$.

---

11 Formally, we also require that the variance of $m$ must be sufficiently small relative to the dispersion of the private signals (that is, relative to $\varepsilon$) to ensure this is still true even if $m$ is close to the boundaries of $(m, \bar{m})$. 

---

28
Throughout Shleifer’s paper it is assumed that \( n \) is large and so the limit applies.

If \( \tilde{m} - m^* < 2\varepsilon \) then we are done. Otherwise we can continue the same argument inductively until \( \tilde{m} \) is reached. That is, we have just shown that for all signals \( m_i \) that are greater than \( m^* \) by an amount \( 2\varepsilon \), firms will never wait before implementing their innovations. We can then repeat the same proof for \( m_i \) that are greater than \( m^* \) by \( 4\varepsilon \) and then \( 6\varepsilon \), etc., until we reach \( \tilde{m} \). We then get a contradiction with our lemma 1, which completes the proof of the proposition.\(^{12}\)

6. Conclusions

In this paper, we have drawn attention to the contrast between the New Economy boom of the 1990s and previous cyclical fluctuations. We argue that this contrast might be explained using Shleifer’s model of implementation cycles. In the first part of the paper, we present some indirect evidence in support of Shleifer’s model. For example, the time series patterns and co-movements of initial public offerings and MFP growth are consistent with the view that innovations are implemented in waves. The association between these two proxies for innovation is quite strong: lagged IPOs help to predict MFP growth, even conditional on lagged MFP growth. This suggests that both these series may be useful proxies for the extent of implementation, and its variation over the business cycle.

The 1990s, however, clearly saw a decline in the volatility of productivity growth. There was a corresponding decline in the volatility of IPOs. Given the similar patterns shown by the two series, we argue that implementation cycles may have weakened in the 1990s. Again, we interpret this in terms of Shleifer’s model. The

\(^{12}\) The arguments used in this proof are in the spirit of the global game literature. In particular, we have followed an approach used by Kim (1996). While Kim’s approach is more general than ours, our proof can be seen as an extension of his Proposition 4, since the game in this paper does not fall directly in the class of games he studies (the game specified by Shleifer’s model does not satisfy Kim’s assumption 1 and exhibits more than two equilibria).
decline in volatility can be explained if, instead of strategic delays, immediate implementation emerged as the equilibrium outcome.

Our theoretical contribution, the second part of the paper, explains this development in the following terms. Recall that, in Shleifer’s model, the timing of implementation of innovations is related to firms’ expectations about future aggregate income. These expectations are self-fulfilling, and business cycles are driven by strategic delays supported by particular expectations. But when we extend Shleifer’s model to incorporate uncertainty about the trend growth rate of the economy, the equilibria with delayed implementation are eliminated, because delay becomes risky. Business cycles with delayed implementation therefore rely on a strong common knowledge assumption, one that may not have been satisfied in the unusual circumstances of the 1990s. We argue that this could explain the reduced volatility in MFP growth and IPOs: uncertainty about the trend growth rate led to immediate implementation as the sole equilibrium outcome.

Although previous researchers have demonstrated the importance of informational assumptions for multiplicity, we have shown that similar arguments apply to a classic model of the business cycle. More ambitiously, we have used this analysis to shed new light on the dynamics of the New Economy in the USA during the 1990s. We argue that introducing uncertainty into Shleifer’s model of business cycles could help to explain some of the most important features of that decade.

References


Chamley, Christophe (2005), Complementarities in Information Acquisition with Short-Term Trades. *Discussion paper, Boston University and CREST*.


Appendix 1. Data sources

Number of IPOs in USA: Updated data on IPOs collected by Jay Ritter, downloaded from website http://bear.cba.ufl.edu/ritter/ipoisr.htm on 3 August 2007. Note that there are small differences between the results in the current paper and those in the working paper version of this research, because of some minor revisions to the IPO data since the previous version was written.

Note that Figure 1, Figure 6, and the first regression in Table 2, are all based on MFP data for 1962-2006. This uses a linked series for MFP, downloaded from the BLS website, 2 August 2007. This linked series is for the private non-farm business sector (excluding government enterprises) and links SIC data for 1948-87 to NAICS data for 1987-2006.

Linked series pre-1987 are not available disaggregated into durables and non-durables sectors. The other MFP series used in the paper were downloaded 29 June 2004, and are no longer updated, because the BLS has switched to the North American Industry Classification System (NAICS). Consistent series on the NAICS basis are not currently available before 1987. We therefore use the following older series that cover the much longer period 1949-2001:

- MFP for private non-farm business sector: series MPU750023(K).
- MFP for manufacturing, non-durables (SIC codes 20-23,26-31): series MPU310003(B).

Appendix 2. Proof of Lemma.

Let $0 \leq j \leq n \cdot (T-1)$ denote the total number of firms who receive innovations at periods 1,...,T-1 and wait until period T before implementing. Let $\Pi_{T/DT-1}(j)$ and $\Pi_{T}(j)$ denote the payoffs from delaying until time $T$ and implementing immediately (respectively) given that exactly $j$ firms wait.
Since (by assumption) $S$ is an equilibrium then the ex-post payoff to the firm from waiting must be higher than that of implementing immediately:

\[
p(j=n(T-l)) \frac{\Pi_T}{D_{T-1}}(n \cdot (T-l)) + p(j=n(T-l)-1) \frac{\Pi_T}{D_{T-1}}(n \cdot (T-l)-1) + \ldots + p(j=0) \frac{\Pi_T}{D_{T-1}}(0) > \\
p(j=n(T-l)) \Pi_1(n \cdot (T-l)) + p(j=n(T-l)-1) \Pi_1(n \cdot (T-l)-1) + \ldots + p(j=0) \Pi_1(0)
\]

Which we can re-arrange as follows:

\[
p(j=n(T-l)) \left[ \frac{\Pi_T}{D_{T-1}}(n \cdot (T-l)) - \Pi_1(n \cdot (T-l)) \right] + \ldots + p(j=kn(T-l)+1) \left[ \frac{\Pi_T}{D_{T-1}}(kn(T-l)+1) - \Pi_1(kn(T-l)+1) \right] > \\
p(j=kn(T-l)) \left[ \frac{\Pi_T}{D_{T-1}}(kn(T-l)) - \Pi_1(kn(T-l)) \right] + \ldots + p(j=0) \left[ \frac{\Pi_T}{D_{T-1}}(0) - \Pi_1(0) \right]
\]

Note that all the expressions in the square brackets – on both sides of the equation – are positive because $\frac{\Pi_T}{D_{T-1}}(j) > \Pi_1(j)$ when $j-k \cdot (T-1)$ and $\frac{\Pi_T}{D_{T-1}}(j) < \Pi_1(j)$ otherwise.

Furthermore, the quantity $\frac{\Pi_T}{D_{T-1}}(j) - \Pi_1(j)$ increases with $j$ (and conversely $\Pi_1(j) - \frac{\Pi_T}{D_{T-1}}(j)$ decreases with $j$). The left-hand side of (10) is therefore smaller than $\phi \left[ \frac{\Pi_T}{D_{T-1}}(n \cdot (T-l)) - \Pi_1(n \cdot (T-l)) \right]$ (because the probabilities sum up to $\phi$). The right-hand side of equation (10) is greater than $(1-\phi) \left[ \Pi_1(kn \cdot (T-l)) - \frac{\Pi_T}{D_{T-1}}(kn \cdot (T-l)) \right]$.

Using these and the inequality (10) we get:

\[
\phi \left[ \frac{\Pi_T}{D_{T-1}}(n \cdot (T-l)) - \Pi_1(n \cdot (T-l)) \right] > (1-\phi) \left[ \Pi_1(kn \cdot (T-l)) - \frac{\Pi_T}{D_{T-1}}(kn \cdot (T-l)) \right]
\]

or $\phi A > (1-\phi)B$ where $A$ and $B$ are both positive. Solving for $\phi$ we get $\phi > \frac{B}{A+B} > 0$. 

34