D Notes for referees

We first show how to derive equation (38). The first step is to divide (37) by (36). We then have:

$$\frac{v}{1-v}\frac{1-s}{s} = \left(\frac{z}{x}\right)^{\alpha}\left(\frac{1-x}{1-z}\right)^{\gamma(1+\lambda)}\left(\frac{b}{a}\right)^{1-\alpha-\beta}\left(\frac{1-a}{1-b}\right)^{(1-\gamma)(1+\lambda)}$$

Using (32) the left-hand side can be rewritten to give:

$$\frac{b}{1-b}\frac{1-a}{a} = \left(\frac{z}{x}\right)^{\alpha}\left(\frac{1-x}{1-z}\right)^{\gamma(1+\lambda)}\left(\frac{b}{a}\right)^{1-\alpha-\beta}\left(\frac{1-a}{1-b}\right)^{(1-\gamma)(1+\lambda)}$$

which can be simplified to:

$$1 = \left(\frac{z}{x}\right)^{\alpha}\left(\frac{1-x}{1-z}\right)^{\gamma(1+\lambda)}\left(\frac{a}{b}\right)^{\alpha+\beta}\left(\frac{1-a}{1-b}\right)^{(1-\gamma)(1+\lambda)-1} \tag{46}$$

We now need to eliminate x and z. Using (34) and (35) we can write

$$\begin{aligned}\left(\frac{z}{x}\right)^{\alpha} &= \left[\frac{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right]^{\alpha}\left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{s}{1-s}\frac{\alpha}{\gamma}}\right)^{\alpha} \\ &= \left(\frac{b}{1-b}\right)^{\alpha}\left(\frac{1-a}{a}\right)^{\alpha}\left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\alpha}\end{aligned} \tag{47}$$

Similarly we can write:

$$\begin{aligned}\left(\frac{1-x}{1-z}\right)^{\gamma(1+\lambda)} &= \left(\frac{\frac{(1-s)\gamma}{s\alpha+(1-s)\gamma}}{\frac{1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}}\right)^{\gamma(1+\lambda)} \\ &= \left(\frac{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}\right)^{\gamma(1+\lambda)}\end{aligned} \tag{48}$$

Combining (47) and (48) we get:

$$\left(\frac{z}{x}\right)^{\alpha}\left(\frac{1-x}{1-z}\right)^{\gamma(1+\lambda)} = \left(\frac{b}{1-b}\right)^{\alpha}\left(\frac{1-a}{a}\right)^{\alpha}\left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\alpha-\gamma(1+\lambda)}$$

Substituting this equation into (46) gives:

$$1 = \left(\frac{a}{b}\right)^{\beta}\left(\frac{1-a}{1-b}\right)^{\lambda}\left(\frac{1-a}{1-b}\right)^{\alpha-\gamma(1+\lambda)}\left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\alpha-\gamma(1+\lambda)}$$

Simplifying further:

$$1 = \left(\frac{a}{b}\right)^{\beta}\left(\frac{1-a}{1-b}\right)^{\lambda}\left(\frac{\frac{1-a}{1-b}\frac{s}{1-s}\frac{\alpha}{\gamma}+\frac{1-a}{1-b}}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\alpha-\gamma(1+\lambda)}$$

which gives:

$$\begin{aligned}
1 &= \left(\frac{a}{b}\right)^{\beta}\left(\frac{1-a}{1-b}\right)^{\lambda}\left(\frac{(1-a)\frac{s}{1-s}\frac{\alpha}{\gamma}+1-a}{b\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1-b}\right)^{\alpha-\gamma(1+\lambda)} \\
1 &= \left(\frac{a}{b}\right)^{\beta}\left(\frac{1-a}{1-b}\right)^{\lambda}\left(\frac{a(\frac{1-a}{a})\frac{s}{1-s}\frac{\alpha}{\gamma}+1-a}{b\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1-b}\right)^{\alpha-\gamma(1+\lambda)} \\
1 &= \left(\frac{a}{b}\right)^{\beta}\left(\frac{1-a}{1-b}\right)^{\lambda}\left[\frac{1-a\left(1-\frac{\alpha}{\gamma}\frac{s}{1-s}\frac{1-a}{a}\right)}{1-b\left(1-\frac{\alpha}{\gamma}\frac{s}{1-s}\frac{1-a}{a}\right)}\right]^{\alpha-\gamma(1+\lambda)}
\end{aligned}$$

as required.

The next part of these notes fills out the details of the derivation of the a^* criterion, discussed in Appendix B and which appears in sections 4 and 8 of the main text. The starting point is the derivative

$$\frac{\partial g}{\partial b}|_{b=a} = \left[\frac{\beta}{a}-\frac{\lambda}{1-a}-\frac{\theta k}{1-ak}\right]g(a)$$

where $k = 1-\frac{\alpha}{\gamma}\frac{s}{1-s}\frac{1-a}{a}$ and $\theta = \alpha-\gamma(1+\lambda)$ all as in Appendix B. The condition for an observed economy to be in a high output equilibrium is that the above expression is greater than zero. Since $g(a) > 0$ (see Appendix B) we require:

$$\frac{\beta}{a}-\frac{\lambda}{1-a}-\frac{\theta k}{1-ak} > 0$$

$$\begin{aligned}
\lambda &< (1-a)\left(\frac{\beta}{a}-\frac{\theta k}{1-ak}\right) \\
\lambda &< (1-a)\left(\frac{\beta}{a}-\frac{(\alpha-\gamma(1+\lambda))k)}{1-ak}\right) \\
\lambda &< (1-a)\left(\frac{\beta}{a}-\frac{(\alpha-\gamma-\gamma\lambda)k}{1-ak}\right) \\
\lambda\left[1-\frac{\gamma k(1-a)}{1-ak}\right] &< \left(\frac{1-a}{a}\right)\beta+\frac{(\gamma-\alpha)(1-a)k}{1-ak} \\
\lambda\left[1-ak-\gamma k(1-a)\right] &< \left(\frac{1-a}{a}\right)(1-ak)\beta+(\gamma-\alpha)(1-a)k \\
\lambda\left[1-\gamma k-ak(1-\gamma)\right]a &< (1-a)(1-ak)\beta+(\gamma-\alpha)(1-a)ak
\end{aligned}$$

Thus we arrive at:

$$\lambda\left[1-\gamma k-ak(1-\gamma)\right]\frac{a}{1-a} < \beta-(\alpha+\beta-\gamma)ak \tag{49}$$

We now consider the first bracketed term, and substitute in for k:

$$\begin{aligned} 1-\gamma k - ak(1-\gamma) &= 1-\gamma+\alpha\frac{s}{1-s}\frac{1-a}{a} - a(1-\gamma) + \frac{\alpha}{\gamma}\frac{s}{1-s}(1-a)(1-\gamma) \\ &= (1-a)\left[1-\gamma+\frac{\alpha}{a}\frac{s}{1-s}+\frac{\alpha}{\gamma}\frac{s}{1-s}(1-\gamma)\right] \\ &= (1-a)\left[1-\gamma+\frac{\alpha}{a}\frac{s}{1-s}\left(1+a\left(\frac{1-\gamma}{\gamma}\right)\right)\right] \end{aligned} \tag{50}$$

Note that this is greater than zero by inspection.

We next simplify the right-hand-side of (49) again substituting in for k:

$$\begin{aligned} \beta-(\alpha+\beta-\gamma)\,ak &= \beta - a\left(1-\frac{\alpha}{\gamma}\frac{s}{1-s}\frac{1-a}{a}\right)(\alpha+\beta-\gamma) \\ &= \beta-\left(a-\frac{\alpha}{\gamma}\frac{s}{1-s}(1-a)\right)(\alpha+\beta-\gamma) \\ &= \beta - a(\alpha+\beta-\gamma)+\frac{\alpha}{\gamma}\frac{s}{1-s}(1-a)(\alpha+\beta-\gamma) \\ &= \beta(1-a)-a(\alpha-\gamma)+\frac{\alpha}{\gamma}\frac{s}{1-s}(1-a)(\alpha+\beta-\gamma) \end{aligned} \tag{51}$$

If we use the results in (50) and (51) to rewrite (49) we obtain:

$$a\lambda\left[1-\gamma+\frac{\alpha}{a}\frac{s}{1-s}\left(1+a\left(\frac{1-\gamma}{\gamma}\right)\right)\right] < \beta(1-a)-a(\alpha-\gamma)+\frac{\alpha}{\gamma}\frac{s}{1-s}(1-a)(\alpha+\beta-\gamma)$$

and hence:

$$\lambda < \frac{\beta(1-a)-a(\alpha-\gamma)+\frac{\alpha}{\gamma}\frac{s}{1-s}(1-a)(\alpha+\beta-\gamma)}{a(1-\gamma)+\alpha\frac{s}{1-s}\left(1+a\left(\frac{1-\gamma}{\gamma}\right)\right)}$$

From this equation it is relatively easy to derive inequality (9) in the text, as follows:

$$\lambda < \frac{\beta+a(\gamma-\alpha-\beta)+\frac{\alpha}{\gamma}\frac{s}{1-s}(\gamma-\alpha-\beta)\,a+\frac{\alpha}{\gamma}\frac{s}{1-s}(\alpha+\beta-\gamma)}{a(1-\gamma)\left(1+\frac{\alpha}{\gamma}\frac{s}{1-s}\right)+\alpha\frac{s}{1-s}}$$

which can be rearranged further:

$$\begin{aligned} \lambda\alpha\frac{s}{1-s}+\lambda a(1-\gamma)\left(1+\frac{\alpha}{\gamma}\frac{s}{1-s}\right) &< \beta+a(\gamma-\alpha-\beta)\left[1+\frac{\alpha}{\gamma}\frac{s}{1-s}\right] \\ &\quad +\frac{\alpha}{\gamma}\frac{s}{1-s}(\alpha+\beta-\gamma) \\ a\left(1+\frac{\alpha}{\gamma}\frac{s}{1-s}\right)(\lambda(1-\gamma)-(\gamma-\alpha-\beta)) &< \beta+\frac{\alpha}{\gamma}\frac{s}{1-s}(\alpha+\beta-\gamma)-\lambda\alpha\frac{s}{1-s} \\ a &< \frac{\beta+\frac{\alpha}{\gamma}\frac{s}{1-s}(\alpha+\beta-\gamma-\lambda\gamma)}{\left(1+\frac{\alpha}{\gamma}\frac{s}{1-s}\right)(\lambda(1-\gamma)-(\gamma-\alpha-\beta))} \end{aligned}$$

which can be rearranged to get equation (9) in the text. Note that to preserve the direction of the inequality, the last line requires $\lambda(1-\gamma)-(\gamma-\alpha-\beta)>0$. Our parameter restriction (8) is sufficient to ensure this.

Finally, we show how to derive equation (45). Starting from (44), we have:

$$\Lambda_F = \frac{Y_n^{'}\left(1+\frac{Y_a^{'}}{pY_n^{'}}\right)}{Y_n\left(1+\frac{Y_a}{pY_n}\right)} = \frac{Y_n^{'}\left(1+\frac{v}{1-v}\right)}{Y_n\left(1+\frac{s}{1-s}\right)}$$

Using the production function for non-agriculture, we can rewrite this as:

$$\Lambda_F = \left(\frac{1-b}{1-a}\right)^{(1-\gamma)(1+\lambda)} \left(\frac{1-z}{1-x}\right)^{\gamma(1+\lambda)} \frac{\left(1+\frac{v}{1-v}\right)}{\left(1+\frac{s}{1-s}\right)}$$

Now we can replace the second bracketed term using equation (48) above, to get

$$\Lambda_F = \left(\frac{1-b}{1-a}\right)^{(1-\gamma)(1+\lambda)} \left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\gamma(1+\lambda)} \left(\frac{1+\frac{v}{1-v}}{1+\frac{s}{1-s}}\right)$$

We now focus on eliminating v from the third bracketed term. We can do this using equation (32). This gives

$$\begin{aligned}
\Lambda_F &= \left(\frac{1-b}{1-a}\right)^{(1-\gamma)(1+\lambda)} \left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\gamma(1+\lambda)} \left[\frac{1+\left(\frac{b}{1-b}\right)\left(\frac{s}{1-s}\right)\left(\frac{1-a}{a}\right)}{\left(1+\frac{s}{1-s}\right)}\right] \\
&= \left(\frac{1-b}{1-a}\right)^{(1-\gamma)(1+\lambda)} \left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\gamma(1+\lambda)} \left[1-s+s\left(\frac{b}{1-b}\right)\left(\frac{1-a}{a}\right)\right] \\
&= \left(\frac{1-b}{1-a}\right)^{(1-\gamma)(1+\lambda)} \left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\gamma(1+\lambda)} \left[1-s\left(1-\left(\frac{b}{1-b}\right)\left(\frac{1-a}{a}\right)\right)\right] \\
&= \left(\frac{1-b}{1-a}\right)^{(1-\gamma)(1+\lambda)} \left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\gamma(1+\lambda)} \left[1-s\left(\frac{a(1-b)-b(1-a)}{a(1-b)}\right)\right] \\
&= \left(\frac{1-b}{1-a}\right)^{(1-\gamma)(1+\lambda)} \left(\frac{\frac{s}{1-s}\frac{\alpha}{\gamma}+1}{\frac{b}{1-b}\frac{s}{1-s}\frac{1-a}{a}\left(\frac{\alpha}{\gamma}\right)+1}\right)^{\gamma(1+\lambda)} \left[1-s\left(\frac{a-b}{a(1-b)}\right)\right]
\end{aligned}$$

which is a simple rearrangement of equation (10) in the text.