Centre for Structural Econometrics

Discussion paper No. 10/06

Optimal Monetary Policy in the Generalized Taylor Economy

Engin Kara
University of Bristol

May 2010

Centre for Structural Econometrics
University of Bristol Department of Economics
8 Woodland Road, Bristol BS8 1TN, UK.
http://www.efm.bris.ac.uk/cse/
econ-cse@bristol.ac.uk

Optimal Monetary Policy in the Generalized Taylor Economy*

Engin Kara[†]

May 7, 2010

Abstract

In this paper, we use the Generalized Taylor Economy *(GTE)* framework to examine the optimal choice of inflation index. In this otherwise standard Dynamic Stochastic General Equilibrium (DSGE) model, there can be many sectors, each with a different contract length. In the *GTE* framework with an empirically relevant contract structure, a simple rule under which the interest rate responds to economy-wide inflation gives a welfare outcome nearly identical to the optimal policy. This finding suggests that it may not be necessary for a well-designed monetary policy to respond to sector-specific inflations.

Keywords: inflation targeting, optimal monetary policy.

JEL: E32, E52, E58.

---

*This paper is a revised version of the 2006 European Central Bank working paper no: 673. For comments and suggestions, I am grateful to two anonymous referees. I would like to thank my Ph.D. supervisor at the University of York, Huw Dixon, for his continuous help and invaluable advice. Part of this paper was completed while I was visiting the Monetary Policy Strategy Division at the European Central Bank. The final version of the paper was written while I was working for the National Bank of Belgium. I also would like to thank Sergio Altimari, Filippo Altissimo, Guido Ascari, Luc Aucremanne, John Fender, Giovanni Lombardo, Klaus Masuch, Neil Rankin, Frank Smets, Leo Von Thadden, Raf Wouters and seminar participants at Birmingham, the National Bank of Belgium, the 2007 Royal Economic Society conference for comments. I am grateful to Michel Juillard for his frequent help with computing

[†]University of Bristol, Economics Department, 8 Woodland Road, Bristol, BS8 1TN. E-mail: engin.kara@bristol.ac.uk.

1 Introduction

The optimal choice of an inflation-index is an important question for policy makers. This paper aims to adress this issue in a model that accounts for the heterogeneity in contract lengths we observe in empirical data. To accomplish this we have used the Generalized Taylor Economy (GTE) (Dixon and Kara (2007, 2010)). The GTE generalizes the simple Taylor model to allow for a distribution of contract lengths in different sectors[1]. An additional advantage of the GTE framework is that it is general enough to represent any distribution of contract lengths, including the one generated by the Calvo model. Dixon and Kara (2007) find that the GTE with a distribution of contract lengths based on the dataset of Bils and Klenow (2004) tracks the U.S. data well.

In this paper, we extend the GTE framework by assuming that each sector is subject to sector-specific productivity shocks. We then consider the design of welfare-maximising inflation-targeting monetary policy rules in a setting where there are multiple sectors, each with a different contract length. We examine the monetary policy implications of alternative assumptions regarding the distribution of contract lengths and explore how to assign weights to different sectors in an optimal inflation index for a central bank to target (i.e. sectoral inflation-targeting). We then compare the performance of the aggregate inflation targeting relative to the sectoral inflation-targeting rule and ask if it is really necessary for a well-designed monetary policy to respond to sector-specific inflation rates.

For this purpose, in our model we derive a utility-based objective function of a central bank by following the procedure described in Rotemberg and Woodford (1998). In doing so, we illustrate the challenge facing the central bank in an environment in which there are many sectors. In particular, we show that welfare in the GTE depends on the variances of the output gap and on the cross-sectional price dispersion. We find that in the GTE

[1] Other papers that emphasize the importance of heterogeneity in inflation and output dynamics include Carvalho (2006), Mash (2004), Sheedy (2007) and Wolman (1999). Carvalho (2006) uses a Multiple Calvo Model (MC). In the MC model, there are many sectors, each with a Calvo style contract (see Dixon and Kara (2007) for a comparison between the MC and the GTE). Mash (2004), Sheedy (2007) and Wolman (1999) use a Generalized Calvo model (GC). The GC generalises the Calvo model to allow the reset probability to vary with the age of the contract. Thus, in this model the hazard rate is duration dependent, rather than constant, as in the Calvo model.

framework, in the presence of sector-specific shocks and nominal rigidities, it is impossible for the central bank to simultaneously stabilize all the objectives; as a result, the first-best allocation cannot be achieved. We then employ Lagrangian methods to determine the optimal policy and use it as a benchmark to evaluate the performance of alternative simple rules.

A main finding of this paper is that in a model with an empirically relevant distribution of contract lengths, a simple rule under which the interest rate responds to economy-wide inflation gives a welfare outcome close to the optimum, which suggests that it may not be necessary for a well-designed monetary policy to respond to sector-specific inflations.

Before we turn to a description of the GTE model, we briefly review the literature on this topic. A rapidly growing literature assesses the question of which inflation index a central bank should target in models that allow for two sectors, such as those studied by Woodford (2003, p. 435-443) and Aoki (2001), or with two countries such as that studied by Benigno (2004). These studies find that targeting economy-wide inflation is not optimal. Instead, they suggest a sectoral inflation targeting rule that puts more weight on the sector in which there is a longer contract. Benigno (2004, p. 295) evaluates the gains from pursuing a sectoral inflation targeting rule at around 0.02% of consumption. This result is consistent with the one we obtain with simple two-sector GTEs. We suggest, however, there is a limitation in studies like these which use models that have only two sectors. Clearly, generating a more realistic case requires going beyond the simple case of two-sector economies. Indeed, we find that in the GTE with an empirically relevant distribution of contract lengths the gains from pursuing sectoral inflation targeting are smaller than what two sector-economies suggest and are virtually zero. In general, we believe that the GTE may be better at capturing the environment facing a central bank.

The remainder of the paper is organised as follows. Section 2 presents the model and Section 3 describes equilibrium dynamics. Section 4 derives a welfare function for a central bank based on the representative household's utility function. Section 5 characterises the optimal policy and Section 6 analyses the implications of various assumptions regarding the distribution of contract lengths and compares the performance of alternative simple inflation-targeting rules. Section 7 summarises our conclusions.

2 The Model

The model we use is the GTE framework of Dixon and Kara (2007). In this otherwise standard DSGE model, there can be many sectors, each with a different contract length. An advantage of the GTE approach is that it is general enough that the model can represent any distribution of contract lengths, including the one generated by the Calvo model. When all the contracts have the same duration in the economy, the model reduces to a standard Taylor model. The exposition here aims to outline the basic building blocks of the model. We first describe the structure of the economy, the behavior of firms (which is standard), the wage-setting decision and monetary policy.

2.1 Structure of the Economy

In the model economy, there is a continuum of firms $f \in [0, 1]$. Corresponding to the continuum of firms f, there is a unit interval of household-unions ($h \in [0, 1]$). The economy is divided into N sectors, indexed by $i = 1...N$. The share of each sector is given by α_i with $\sum_{i=1}^{N} \alpha_i = 1$. Within each sector i, each firm is matched with a firm-specific union($f = h$): there are i equally sized cohorts $j = 1...i$ of unions and firms[2][3]. Each cohort sets the wage which lasts for T_i periods: one cohort moves each period. The share of each cohort j within the sector i is given by $\lambda_{ij} = \frac{1}{T_i}$ where $\sum_{j=1}^{T_i} \lambda_{ij} = 1$. The longest contracts in the economy are N periods.

All other basic elements of the model are standard New Keynesian. A typical firm produces a single differentiated good. The production of intermediate goods requires labor as the only input. The final consumption good is a constant elasticity of substitution (CES) aggregate over the differentiated intermediate goods. The representative household derives utility from consumption and leisure. The government conducts monetary policy and provides subsidies that offset any distortions in the steady state. The

[2] This assumption means that there is a firm- specific labour market. The implications of the firm-specific labour market assumption on inflation dynamics are well known (see for example Woodford (2003, p. 163-178) Dixon and Kara (2007) and Edge (2002)).

[3] In this model, a firm and a household can be thought of as the same entity, as each household h is matched with firm f. Thus, if we were to assume that wages are flexible while goods prices are sticky, the equilibrium conditions would be the same as the case in which wages are sticky while goods prices are flexible.

subsides are financed by lump-sum taxes.

2.2 Firms

A typical firm in the economy produces a differentiated good which requires labour as the only input, with a CRS technology represented by

$$Y_{ft} = A_{it} L_{ft} \tag{1}$$

where $a_{it} = \log A_{it}$ is a productivity shock in sector i and follows the $AR(1)$ process: $a_{it} = \rho_i a_{it-1} + \varepsilon_{it}$. $f \in [0, 1]$ is firm specific index. Differentiated goods $Y_t(f)$ are combined to produce a final consumption good Y_t. The production function here is CES and corresponding unit cost function P_t

$$Y_t = \left[\int_0^1 Y_{ft}{}^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, P_t = \left[\int_0^1 P_{ft}^{1-\theta} df \right]^{\frac{1}{1-\theta}} \tag{2}$$

The demand for the output of firm f is given by

$$Y_{ft} = \left(\frac{P_{ft}}{P_t} \right)^{-\theta} Y_t \tag{3}$$

The firm chooses $\{P_{ft,} Y_{ft}, L_{ft}\}$ to maximize profits subject to (1, 3), yields the following solutions for price, output and employment at the firm level given $\{Y_t, W_{ft}, P_t\}$.

$$P_{ft} = \frac{\varepsilon}{(1+\tau)} \frac{W_{ft}}{A_{it}} \tag{4}$$

$$Y_{ft} = \left(\frac{\varepsilon}{(1+\tau)} \right)^{-\theta} \left(\frac{W_{ft}}{A_{it} P_t} \right)^{-\theta} Y_t \tag{5}$$

$$L_{ft} = \left(\frac{\varepsilon}{(1+\tau)} \right)^{-\theta} \left(\frac{1}{A_{it}} \right) \left(\frac{W_{ft}}{A_{it} P_t} \right)^{-\theta} Y_t \tag{6}$$

Where $\varepsilon = \frac{\theta}{\theta-1}$ measures the markup. τ denotes a subsidy to employment, which reflects our assumption that the labour income is subsidized in order to eliminate monopolistic distortion[4]. Therefore, in the steady state of the

[4]Similiar assumption can be found in Gali (2003), Huang and Liu (2005) and Erceg and Levin (2006).

model prices are equal to marginal cost, as in the perfectly competitive economy. Price is an effective markup (adjusted for subsidy) over marginal cost, which depends on the wage rate (W_{ft}) and the sector specific productivity shocks.

2.3 Household-Unions and Wage Setting

The representative household h has a utility function given by

$$U_h = E_t \left[\sum_{t=0}^{\infty} \beta^t \left[U(C_{ht}) + V\left(1 - H_{ht}\right) \right] \right] \tag{7}$$

where C_{ht}, H_{ht} are household $h's$ consumption and hours worked respectively, t is an index for time, $0 < \beta < 1$ is the discount factor, and $h \in [0, 1]$ is the household specific index.

The household's budget constraint is given by

$$P_t C_{ht} + \sum_{s^{t+1}} Q(s^{t+1} \mid s^t) B_h(s^{t+1}) \leq B_{ht} + (1+\tau) W_{ht} H_{ht} + \Pi_{ht} - T_{ht} \tag{8}$$

where $B_h(s^{t+1})$ is a one-period nominal bond that costs $Q(s^{t+1} \mid s^t)$ at state s^t and pays off one dollar in the next period if s^{t+1} is realized. B_{ht} represents the value of the household's existing claims given the realized state of nature. W_{ht} is the nominal wage, Π_{ht} is the profits distributed by firms and $W_{ht} H_{ht}$ is the labour income. τ denotes the fixed rate at which labour income is subsidized[5]. Finally, T_t is a lump-sum tax.

The first order conditions derived from the consumer's problem are as follows:

$$u_{ct} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} u_{ct+1} \right) \tag{9}$$

$$\sum_{s_{t+1}} Q(s^{t+1} \mid s^t) = \beta E_t \frac{u_{ct+1} P_t}{u_{ct} P_{t+1}} = \frac{1}{R_t} \tag{10}$$

[5] Note that the subsidy is a policy variable and we assume that the government sets it equal across consumers. Therefore, τ does not have the subscript h.

$$X_{it} = \frac{\varepsilon}{(1+\tau)} \left[\frac{E_t \sum_{s=0}^{T_i - 1} \beta^s \left[V_L \left(1 - H_{it+s}\right) \left(K_{t+s}\right)\right]}{E_t \sum_{s=0}^{T_i-1} \beta^s \left[\frac{u_c(C_{t+s})}{P_{t+s}} K_{t+s} \right]} \right] \tag{11}$$

where

$$K_t = \left(\frac{\theta}{1-\theta}\right)^{-\theta} P_t^{-\theta} Y_t$$

Equation (9) is the Euler equation. Equation (10) gives the gross nominal interest rate. Equation (11) shows that the optimal wage or the reset wage in sector i (X_{it}) is a constant "mark-up" (given by ε which is adjusted for subsidy) over the ratio of marginal utilities of leisure and marginal utility from consumption within the contract duration $s = t...t + T_i - 1$[6]. K_t collects all the terms which the union treats exogenous. As mentioned before, τ denotes a subsidy to household-unions. Therefore, the steady state of the model satisfies the efficiency condition that the marginal rate of substitution equals the real wage, as in a perfectly competitive economy. Note that the index h is dropped in equations (9) and (11), which reflects our assumption of complete contingent claims markets for consumption and implies that consumption is identical across all households in every period ($C_{ht} = C_t$).

2.4 The Government

The labour income subsidy is financed by lump sum taxes. In particular, we assume that

$$T_t = \tau W_t H_t \tag{12}$$

We do not specify a particular policy at this stage since our objective is to find an optimal monetary policy in the economy. However, given any monetary policy, an equilibrium can be computed.

3 Equilibrium Dynamics

We consider an equilibrium in which each sector i is identified by the contract length T_i and a household-union in each sector is identified by the time

[6]Note that when $T_i = 2$, equation (11) reduces to the first order condition in Ascari (2000).

at which it can set a new wage. We follow the standard approach of log-linearizing around the steady state of the model. We follow the notational convention that lower-case symbols represent log-deviations of variables from the steady state and variables with asterisks denote the equilibrium value of variables under flexible wages.

3.1 Log-linearized Economy

We now turn to characterize the sticky wage equilibrium of the economy. We render nominal variables such as wage level and price level as stationary by reexpressing them in terms of log-devations from the aggregate price level. For example, $\bar{x}_{it}$ and $\bar{p}_{it}$ denote the logarithmic deviation of the reset wage and price level in sector i from the aggregate price level, respectively.

The linearized wage decision equation (11) for sector i is given by

$$\bar{x}_{it} = \sum_{s=1}^{T_i-1} \Phi_k \pi_{t+s} + \frac{1}{\sum_{s=0}^{T_i-1} \beta^s} \left[\sum_{s=0}^{T_i-1} \beta^s \left[\gamma \tilde{y}_{t+s} + \gamma y^*_{t+s} - \delta a_{it+s} \right] \right] \tag{13}$$

with

$$\gamma = \frac{(\eta_{cc}+\eta_{ll})}{(1+\theta\eta_{ll})} \quad \Phi_k = \frac{\sum_{k=s+1}^{T_i-1} \beta^k}{\sum_{k=s}^{T_i-1} \beta^k} \quad \delta = \frac{\eta_{ll}(1-\theta)^7}{(1+\theta\eta_{ll})} \tag{14}$$

Where $\tilde{y}_t = y_t - y^*_t$ is the gap between actual output, y_t and the flexible-price equilibrium output level y^*_t, π_t is the aggregate inflation rate and θ is the elasticity of substitution of consumption goods. $\eta_{cc} = \frac{-U_{cc}C}{U_c}$ is the parameter governing risk aversion, $\eta_{ll} = \frac{-V_{ll}L}{V_l}$ is the inverse of the labour elasticity.

In each sector i, the sectoral inflation is related to the wage level in sector i through a relation of the form

$$\sum_{s=1}^{T_i-1} \sum_{j=s}^{T_i} \lambda_{ij} \pi_{it-s-1} = \sum_{j=1}^{T_i-1} \lambda_{ij} \left[\bar{x}_{it-j} - \bar{p}_{it-j-1} - a_{it} \right] \tag{15}$$

Where $\bar{p}_{ijt} = \bar{x}_{it-j} - a_{it}$ is the logarithmic deviation of the price level in sector i cohort j from the aggregate price level.

[7] When $\eta_{cc} = 1$, $\delta = (\gamma - 1)$.

By using the fact that the linearized price level in the economy is the weighted average of the ongoing prices in the economy, we obtain the following identity:

$$\sum_{i=1}^{N} \alpha_i \bar{p}_{it} = 0 \tag{16}$$

where $\bar{p}_{it}$ can also be expressed as

$$\bar{p}_{it} = \bar{p}_{it-1} + \pi_{it} - \pi_t \tag{17}$$

Using these identities, aggregate inflation can be expressed as

$$\pi_t = \sum_{i=1}^{N} \alpha_i \left(\bar{p}_{it-1} + \pi_{it} \right)$$

which implies that aggregate inflation depends on both sectoral lagged relative prices and inflation levels.

Since nominal rigidities arise due to the assumption of staggered wages and prices are perfectly flexible in the model, wage inflation is given by

$$\pi_{it}^{w} = \pi_{it} + \Delta a_{it} \tag{18}$$

where π_{it}^{w} denotes the wage inflation in sector i and $\Delta a_{it} = a_{it} - a_{it-1}$.

Log-linearizing (9) gives

$$y_{it} = -\theta \bar{p}_{it} + y_t \tag{19}$$

Using the log-linearized version of the household's intertemporal Euler equation (9) and subtracting the flexible-wage version, we obtain the Euler equation in terms of output gap, which is given by

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \eta_{cc}^{-1} (r_t - E_t \pi_{t+1} - rr_t^*) \tag{20}$$

where $rr_t^* = r_t^* - E_t \pi_{t+1}^* = E_t y_{t+1}^* - y_t^*$ denotes the real interest rate when prices are flexible. r_t^*, π_{ft}^* and y_t^* denote the nominal interest rate, the inflation rate and the output level when prices are flexible, respectively.

The solution for y_t^* is given by

$$y_t^* = \frac{(1 + \eta_{ll})}{(\eta_{ll} + \eta_{cc})} a_t \tag{21}$$

where $a_t = \sum_{i=0}^{N} \alpha_i a_{it}$ is the weighted average of productivity across all the sectors in the economy. This equation implies that the natural level of output is a weighted average of productivity across all the sectors in the economy.

When all wages are completely flexible, the wage decision does not depend on the inflation level, as wages can adjust every period. Moreover, by definition the output gap is zero. Therefore, the wage setting rule (13) reduces to

$$\bar{x}_{it}^{*} = \gamma y_t^{*} - \delta a_{it} \tag{22}$$

Thus, the relative prices when prices are fully flexible are given by

$$\bar{p}_{it}^{*} = \gamma y_t^{*} - (1+\delta) a_{it} \tag{23}$$

This equation implies that relative prices fluctuate in response to shocks.

4 The welfare function for the *GTE*

This section generalises the analysis of Rotemberg and Woodford (1998) and derives a utility-based objective function of a central bank to provide a benchmark for evaluating the performance of alternative inflation-targeting monetary policy rules. The welfare function is given by the sum of all households' utility function:

$$W_t = \sum_{t=0}^{\infty} \beta^t U_t$$

where

$$U_t = \left[U(C_t) + \sum_{i=1}^{N} \sum_{j=1}^{T_i} \alpha_i \lambda_{ij} V\left(1 - H_{ijt}\right) \right]$$

Consumption is identical across all households in every period, reflecting our assumption of complete contingent; but labour can vary across cohorts.

As shown in the appendix, by taking the second-order logarithmic approximation to this utility function around a steady state, the welfare function can be expressed as

$$W = \sum_{t=0}^{\infty} \beta^t U_t = -\frac{U_c(C)C}{2} \sum_{t=0}^{\infty} \beta^t L_t \tag{24}$$

where the loss function is given by

$$L_t = \left[(\eta_{cc} + \eta_{ll})\tilde{y}_t^2 + \theta \sum_{i=1}^{N} \sum_{j=1}^{T_i} \alpha_i \lambda_{ij} \tilde{p}_{ijt}^2 \right]$$

where $\lambda_{ij} = \frac{1}{T_i}$ and $\tilde{p}_{ijt} = \bar{p}_{ijt} - \bar{p}_{ijt}^{*}$ denotes the relative price gap. This expression implies that welfare loss depends on the variance of the output gap and on the cross-sectional price dispersion. Sectors differ based on their contract length and their budget share. When there is only one type of contract length in the economy, the function reduces to the welfare function in a standard one sector model, as in Paustian (2005).

Note that this welfare function is roughly analogous to those obtained under the popular specification of the Calvo model. One main difference between the GTE and the Calvo model is that in the Calvo model the wage setters do not know how long the wage will last. In that model, each period, a randonly chosen fraction ω of cohorts, adjust their wages. Note that all resetting cohorts set the same wage. Therefore, in the Calvo model in each period there is only one reset wage ($N = 1$). In contrast, in the GTE there is a distribution of sector specific reset wages, $\bar{x}_{it}$, in each period. However, the Calvo process can be described in deterministic terms at the aggregate level because there the firm-level randomness disappears. To put it differently, at the aggregate level, the precise identity of individual firms does not matter; what matters is the proportion of firms that set contracts of particular lengths at particular times. The steady-state cross-section of contract ages can be described by the proportion λ_{ij} of cohorts surviving at least j periods: $\lambda_{ij} = \omega (1-\omega)^{j-1} : j = 1..\infty$. Therefore, the welfare function for the Calvo model is given by

$$W = -\frac{U_c(C)C}{2} \left[(\eta_{cc} + \eta_{ll})\tilde{y}_t^2 + \theta \sum_{j=1}^{T_i} \lambda_{ij} \bar{p}_{ijt}^2 \right]$$

where

$$N = i = 1 \quad T_i = \infty \quad \lambda_{ij} = \omega (1-\omega)^{j-1}$$

Woodford (2003) shows that the welfare costs of cross-sectional price dispersion can be summarised in terms of variability of inflation in the Calvo model. In contrast, in the GTE, this is not the case: the welfare costs of cross-sectional dispersion cannot be summarised in terms of variability of inflation and must be given explicitly in terms of variances of relative prices. This is mainly because in the GTE, there is a distribution of sector-specific reset wages in each period.

A comparison with the welfare function derived in Aoki (2001) is useful. The loss function in Aoki (2001) is different from ours beacuse of slightly different model structures. If we were to adopt the modelling approach of Aoki (2001), then loss function in the new model would depend on the variability of the output gap, the dispersion of prices across firms in sector i $\left(\text{i.e.}\theta \sum_{i=1}^{N} \sum_{j=1}^{T_i} \alpha_i \lambda_{ij} \left(x_{ijt} - p_{it}\right)^2\right)$ and the variability of the sectoral relative price gap (i.e. the deviation of sectoral relative price from its efficient level). The difference arises from the fact that in the GTE, there is a continuum of firms that produce differentiated goods, which are then combined to produce the final consumption good. The production function is Dixit-Stiglitz. Therefore, the demand for an individual firm depends only on its own price and the general price index (see equation (3)). While we divide the unit interval into segments corresponding to sectors and cohorts within sectors, this need not reflect any objective factor in terms of sector or cohort specific aspects of technology or preferences. Sectors are composed of firms that have the same contract length and are hit by the same shocks. Thus, there is no sense of location and α_i and λ_{ij} can best be thought of as simply measures of sector and cohort size. As noted in Dixon and Kara(2007), this is an important property, that allows it to be demonstrated that a Calvo economy can be represented by a GTE. However, Aoki's model has a typical two-sector structure. If we were to follow the Aoki approach, then we would have to assume that there are two sectors, each producing a different consumption goods. Within each sector, there are many firms, represented by a continuum over the unit interval. The sector's output is a Dixit-Stiglitz aggregate of the firms' output. The final consumption is a Dixit-Stiglitz aggregate over the goods in the two sectors of the economy. Therefore, the demand for an individual firm in sector i depends on its own price and the general price level in sector i. The demand for sector i depends on the general price level in sector i and the general price level in the economy. Therefore, in such a setting the welfare function depends on the dispersion of prices across sectors and firms as well as the variance of the output gap. Thus, Aoki used a model and, therefore, a loss function different from ours. Nonetheless, as we will show later, his conclusions carry over to our setting. Another related paper is that by Erceg and Levin (2006). Erceg and Levin (2006) assume that the sectors are completely independent from each other. Therefore, the loss function in Erceg and Levin (2006) depends on the volatility of the sectoral output gap and on the dispersion of prices across firms in each sector.

5 Optimal Monetary Policy in the *GTE*

We turn now to examining the issue of optimal monetary policy. We begin with considering if the Pareto optimal allocation can be achieved in the *GTE*. As the welfare function shows, an equilibrium allocation is Pareto optimal in the *GTE* only if the relative prices and output gap is zero in every period, that is $\tilde{y}_t = \tilde{p}_{it} = 0$ for all t.

In the *GTE*, it is impossible to satisfy all of the stabilisation objectives at the same time: therefore, Pareto optimal allocation is not attainable. To see this, assume that the economy starts in a steady state in which all gaps are zero. When the sector specific productivity shocks hit the economy in period t, in the efficient equilibrium, both output and relative prices fluctuate in response to the shocks. The central bank cannot stabilize both of them at the same time. This is most easily seen by considering the evolution of the output level in sector i, where $1 < i \leq N$. For expositional simplicity, without loss of generality, let $\eta_{cc} = 1$. Then, the relative prices when wages are fully flexible is $\bar{p}^*_{it} = \gamma(a_t - a_{it})$ and, therefore, the output gap in sector i is given by

$$\tilde{y}_{it} - \tilde{y}_t = -\theta \left(\bar{p}_{it} - \gamma(a_t - a_{it})\right) \tag{25}$$

This equation makes clear that in the presence of asymmetric productivity shocks, closing the gap in sector i requires that prices adjust. Thus, in the GTE, in the presence of asymmetric productivity shocks, in the efficient equilibrium both output and relative prices fluctuates in response to shocks, creating a trade-off for the central bank.

In contrast, when all shocks are identical, the Pareto optimal allocation can be achieved. This is because when shocks are identical, in the efficient equilibrium only output fluctuates in response shocks. Consider the scenario in which the central bank implements a policy that closes all of the sectoral output gaps. A zero sectoral output gap would induce firms to choose a relative price of zero. Thus, in this case, stabilising output automatically stabilises prices.

Recent work by Blanchard and Gali (2007) shows that a policy trade-off between price stability and output gap stability emerges if real wage rigidities are introduced into the standard model. It is difficult to draw a precise isomorphism between the model in this paper and that in Blanchard and Gali (2007), as Blanchard and Gali (2007) employ the ad hoc device of real wage rigidities. Therefore, the mechanism at work in this paper may suggest a more plausible mechanism to generate a trade-off than the one proposed by Blanchard and Gali (2007).

5.1 Second-Best Optimal Policy

As the discussion in the previous section reveals, in the GTE it is not possible to achieve the first-best allocation. Accordingly, in this case one must consider the second-best optimal policy. We use Lagrangian methods to characterize the second best optimal monetary policy. In particular, we compute the optimal policy that can be obtained by maximizing the welfare level defined in (24) subject to the equilibrium conditions (13) - (18). While this is an useful reference, as discussed in Huang and Liu (2005), it is difficult to implement, as it requires the knowledge of leads and lags of the inflation rates and the output gap. Therefore, we use the central bank's first order conditions along with the equilibrium conditions for the model to solve and calculate the level of welfare under optimal monetary policy. We then use this level as a benchmark to compare the performance of alternative simple rules, with the coefficients in front of the targeting variables are chosen to maximize welfare. These are considered as feasible and effective tools to implement monetary policy.

5.2 Alternative Simple Policy Rules

We assume that the central bank has adopted an inflation-targeting regime. We consider two policy rules. The first rule is a sectoral inflation-targeting rule under which the central bank responds to an appropriately weighted average of the sectoral wage inflation rates[8]. In this case, the interest rate rule takes the following form:

[8]Given the fact that nominal rigidities in our model arise due to the assumption of sticky wages, and therefore wage stickiness plays a crucial role in determining the welfare

$$r_t = \sum_{i=1}^{N} \phi_i \pi_{it}^w$$

where π_t^w represents the wage inflation in sector i. The reaction coefficients in front of the targeting variables ($\phi-$coefficients) are optimally chosen. The weight that sector i receives in the optimal inflation index is given by $\frac{\phi_i}{\sum_{i=1}^{N} \phi_i}$.

The second rule is an aggregate inflation-targeting one under which the central bank responds to aggregate wage inflation. This rule takes the following form:

$$r_t = \phi_a \pi_t^w$$

where $\pi_t^w = \sum_{i=1}^{N} \alpha_i \pi_{it}^w$ is aggregate wage inflation. The reaction coefficient, ϕ_a, is optimally chosen.

5.3 Choice of Parameters and Computation

We use a discount factor β of 0.99 which corresponds to the annual real interest rate in the steady state of 4%. As discussed in Dixon and Kara (2007), we set $\eta_{ll} = 4.5$, implying that intertemporal labour supply elasticity, $1/\eta_{ll}$, is 0.2. We set θ= 6. This measures the elasticity of substitution between goods. We set the relative aversion in consumption,η_{CC}, as unity. Finally, we set the $\rho_i = 0.95$ and the standard deviations of innovations to productivity shocks $\sigma_i = 0.02$, a standard assumption in the literature (e.g. Huang and Liu (2005)).

5.3.1 Computation

All calculations are performed using Dynare version 3.06 (see Juillard (1996)). To compute the optimal weights in the optimal inflation index, we numerically minimise the welfare loss with respect to the parameters in the mone-

cost of business cycles, here we consider the cases in which the inflation measure that the central bank chooses to target is the wage inflation rather than the price inflation, as is suggested by Erceg, Henderson and Levin (2000). In another exercise that we do not report here, we consider policy rules that respond to the price inflation. We find that, not surprisingly, the performance of the wage inflation rule is always better compared to the price inflation rule.

tary policy rule ($\phi-$coefficients), subject to the equilibrium conditions and additional non-negativity constraints: $\phi_i \geqslant 0$[9].

6 Results

We now proceed to examine how the policy rules perform under alternative assumptions regarding the distribution of contract lengths. To do this, we allow for different distributions of contracts in the GTE framework, and for each case, we evaluate the performance under the alternative simple rules in comparison to that under optimal policy. In order to illustrate the nature of the problem faced by the monetary authority in the model and the implications of sectoral heterogeneity for policy design, we start with the simple case of two-sector GTEs, a common assumption in the literature. We then explore monetary policy implications of the model that assumes a wider range of contract lengths, consistent with the evidence provided by recent studies (e.g. Bils and Klenow (2004)). Throughout the paper, welfare levels (W) are expressed in terms of the equivalent percentage decline in steady state consumption, which can be obtained by dividing W by U_cC (and multiplying by 100). Welfare levels under optimal policy corresponds to that under the second-best optimal policy discussed in section 5.1.

6.1 Simple *GTE*s

We start by considering the simple case of two-sector $GTEs$. In particular, we let the assumed contract length in sector 2 vary between complete flexibility ($T_2 = 1$) and 8-period contracts ($T_2 = 8$), while assuming $T_1 = 2$. In each case, we assume that sectors have equal shares and calculate the welfare losses under different policy regimes: the sectoral inflation-targeting rule, the aggregate inflation-targeting rule and the optimal policy. Figure 1 reports the welfare losses under the three policy schemes; Figure 2 plots the optimal weights on Sector 2 in the optimal inflation index when the central bank adopts the sectoral inflation-targeting.

[9] To do so, we use the optimization routine "fminsearch" in Matlab 7. The tolerance level is set at $1e-6$. For robustness, we started the routine from several points. The results reported in the paper appear robust.

Figure 1 here

Figure 2 here

We begin by comparing the performance of the alternative rules. As Figure 1 shows, the sectoral inflation-targeting rule outperforms the aggregate inflation targeting rule and yields a welfare outcome nearly identical to the optimal policy, except in the case where sectors have identical contract lengths. Not surprisingly, when the sectors have identical contract lengths, the aggregate inflation-targeting rule performs as well as the sectoral inflation-targeting rule. Figure 2 shows that if the central bank adopts the sectoral inflation-targeting rule, the optimal rule places the largest weight on the sector in which the prices are stickier. Moreover, the optimal weight that the long-contract sector receives increases with the contract length in that sector. That is, the longer the contract duration in sector 2, the higher weight the sector should receive in the optimal inflation index. Thus, a simple rule that puts more weight on the sector that has longer contracts brings the welfare level not too far from the optimal policy. These results are in line with the findings of Benigno (2004) and Woodford (2003). In addition, if one sector has fully flexible wages, then the central bank should react to the sector that has sticky wages, a practice in line with the findings of Aoki (2001).

In order to understand these results, first note that price stickiness dampens the effect of a shock on prices. In other words, when prices are sticky, firms cannot change their prices as much as they would when prices are flexible. This sluggish adjustment means that a small change in prices implies a large change in firms' optimal prices. This is because firms, when resetting their prices, take into account the fact that they will have to charge the same price during the contract length. Since there is a trade-off between price stability and the output gap stability, the large movements in firms' price require large movements in the output gap to control price stability.

In the case of a two-sector economy, an inflation-targeting central bank can offset the disruptive effect of the stickier sector by putting a greater weight on that sector. In addition, as discussed in Dixon and Kara (2007), in the GTE model the presence of the longer contracts influences the wage-setting behavior of short-term contracts via the aggregate price level. In other words, there is a spillover effect from the sluggish long-contract sectors to the short-contract sectors via the aggregate price level. Given the fact that prices in the long-contract sector will adjust sluggishly in response to technology shocks means that prices in the short-contract sector will also adjust sluggishly. As a result, the presence of longer contracts would be even more disruptive. A policy rule that can minimize the disruptive effect of the long-contract sector can also minimize the disruptive effect of the longer-contract sector on the aggregate price level and hence on the short-contract sector. In contrast, the aggregate inflation targeting rule is less effective in controlling price stability because this rule implicitly puts too much emphasis on the short-contract sector. This occurs because the target weights under the aggregate inflation-targeting scheme are not optimally chosen and are the sectoral weights. Reduced target weight of the long-contract sector under the aggregate inflation-targeting rule means that this policy cannot offset the disruptive effect of the long-contract sector as much as the sectoral inflation-targeting can. As a consequence, under the aggregate inflation-targeting scheme, aggregate inflation will move sluggishly. As noted above, higher degree of persistence means larger movements in the output gap are required to control price stability. Larger fluctuation under the aggregate inflation-targeting scheme leads to a significant deterioration in social welfare. Figure 3 confirms this intuition. Reported there are the standard deviations of the output gap under the aggregate inflation-targeting rule and the sectoral inflation-targeting rule. As the table shows, the standard deviation of the output gap under the aggregate inflation-targeting rule can be several times larger than that under the sectoral inflation-targeting rule.

Figure 3 here

Finally, we can ask the question: how much welfare gain is derived from moving to the sectoral inflation-targeting rule from the aggregate inflation-targeting rule. To provide an answer, we consider the case in which the welfare gains from switching to the sectoral inflation-targeting rule, as Figure 1 indicates, is largest: a two sector economy in which sector 1 wages adjust every period and sector 2 wages adjust every two periods. The welfare gain from switching the sectoral inflation targeting policy is around 0.02% of consumption. If we assume that in sector 2 wages adjust every 8 periods rather than every 2 periods and hold the other factors constant, the gain goes up to 0.03%. Note that the scale of welfare gains is in line with those in Benigno (2004).These numbers, however, do not seem large. However, there are gains in following the sectoral inflation targeting policy. In fact, given that Pareto optimal allocation can be achieved in these cases, the sectoral inflation targeting rule eliminates the welfare cost associated with price stickiness.

6.2 Distribution of Contract Lengths

We have thus far considered simple GTEs, in which there are only two sectors. We now examine the implications for optimal monetary policy design of allowing a wider range of contract lengths in order to see if the conclusion based on two sectors carries over to a more realistic, empirically relevant case. We will consider some special GTEs that assume more than two sectors.

6.2.1 Taylor's US Economy

In this section, we use the study by Taylor (1993) to calibrate the U.S. economy. Taylor calibrates the U.S. economy as $\mathbf{T} = (1, 2, 3, 4, 5, 6, 7, 8)$, with sector shares of $\alpha_1 = 0.07$, $\alpha_2 = 0.19$, $\alpha_3 = 0.23$, $\alpha_4 = 0.21$, $\alpha_5 = 0.15$, $\alpha_6 = 0.08$, $\alpha_7 = 0.04$, $\alpha_8 = 0.03$. Note that the largest sector is $3-$period contracts; the three contract lengths $(3, 4, 5)$ each have about 20%, with a fat tail of longer contracts (as many 7- and 8-quarter contracts as 1-quarter contracts). The distribution is plotted in Figure 4. The average contract length in this economy is 3.6 periods. Table 1 displays the welfare losses under the different policy regimes in Taylor's U.S. economy: the aggregate-inflation targeting rule and the sectoral-inflation targeting rule. Figure 4 reports the optimal weights that sectors receive in the optimal index. Two important results emerge from this experiment:

Table 1 here

Figure 4 here

First, the welfare gain in switching to the sectoral inflation-targeting rule is even smaller than what two-sector economies suggest. In Taylor's US economy, as evident from Table 1, the welfare gain is only 0.002% of steady state consumption, which is about a tenth of what it is in two-sector economies. Second, if the central bank follows a sectoral inflation-targeting rule, then the optimal rule does not necessarily place the largest weight on the inflation rates where contract lengths are the longest.

Table 2 here

The first result noted here raises a natural question: why is the welfare gain of moving from aggregate inflation-targeting to sectoral inflation-targeting quantitatively so much smaller in a model with distribution of contract lengths than in a model with two sectors? The main reason for this result is that in a model that accounts for the heterogeneity of contract durations that we observe in empirical data, fluctuations in the output gap are less costly compared to those in a model with only two sectors. A greater degree of nominal rigidity in an economy with a distribution makes it more important to control price stability and that reduces the relative weight of the output gap term in the loss function. As Table 2 shows, as in the case of two-sector economies, the volatility of the output gap is indeed lower under the sectoral rule compared with volatility under the aggregate inflation-targeting rule. However, since controlling price stability is more important in Taylor's U.S. economy due to a higher degree of price dispersion, the output gap term receives less weight in the loss function. As a result, deterioration in social welfare caused by a larger degree of fluctuations in the output gap under the aggregate inflation-targeting scheme in Taylor's economy is small.

Figure 5 here

Perhaps the importance of this channel can be best illustrated by varying the share of the flexible sector in the economy from 0.07 to 0.5 and reallocating the remaining shares to the other sectors according to their relative importance in the sectoral index. Clearly, as the flexible-sector share increases, the longer-contract sectors receive less weight and, as a consequence, the degree of nominal rigidity in the economy decreases. Figure 5 reports the welfare losses under the two policy regimes when the share of the flexible-contract sector varies from 0.07 to 0.5. As the figure shows, the welfare gain of moving from aggregate-inflation targeting to sectoral-inflation targeting increases as the longer contract sectors become less important in the economy and the economy becomes more flexible. This is because a lower degree of nominal rigidity makes it less important to control price stability, increasing the relative weight that the variance of the output gap term receives in the loss function.

The second result reported above suggests that the shape of the distribution is crucial when constructing an optimal inflation index for a central bank to target. As Figure 4 shows, the longer contracts, more important in the optimal index, get higher weights. More specifically, the optimal weights are less than the corresponding sectoral weight for the sectors that have relatively short contracts; but as the contract length increases, the sectors with longer contracts start to get higher weights compared with the sectoral weights. As contract length further increases, the optimal weights decline because the sectoral shares fall. In other words, in Taylor's U.S. economy, the most common durations $--3,4$ and $5--$ each have about 20% share and there are only a few firms that belong to the longest (8-period) contract. The behavior of these latter firms simply does not matter as much for optimal policy. In fact, this intuition carries over to the case in which there are only two sectors in the economy. Consider, for example a case in which sector 1 has a 2-period contract with a share of 95%, while sector 2 has an 8-period contract with a share of 5%. In this case, the sectoral inflation-targeting rule places the largest weight on the short-contract sector. The optimal weight of the 2-period contract sector is around 60%, while the optimal weight of the 8-period contract sector is 40%. This result clearly shows that, even if one assumes that a two-sector economy closely approximates either the U.S. economy or the Euro-Area, the optimal weight that a sector receives depends also on the share of a sector in the economy. A failure to take into account a sector's share, when designing an inflation index for a central bank to target, can thus result in policies that are suboptimal. Although this finding may seem obvious, the perception in the existing literature that the sector that has the longest contract receives the largest weight in the optimal inflation index arises from the assumption that the sectors have equal sizes.

6.2.2 Bils-Klenow GTE (BK-GTE)

Finally, we consider the implications of the Bils-Klenow GTE (BK-GTE), based on the Bils and Klenow (2004) data set. This is for *price* data, but we use it as an illustrative data set. The data are derived from the US Consumer Price Index data collected by the *Bureau of Labor statistics*. The period covered is 1995-7, and the 350 categories account for 69% of the CPI. The data set gives the average proportion of prices changing per month for each category. We assume that these data are generated by a simple Calvo process within each sector. We generate the distribution of durations for that category using Dixon and Kara (2006). We then sum over all sectors using the category weights. For computational purposes, the distribution is truncated at $N = 20$, with the 20-period contracts absorbing all of the weights from the longer contracts[10]. The distribution in terms of quarters is plotted in Figure 6. The mean contract length is 4.4 quarters. There is a very long tail, indicating some very long contracts: over 3% of weighted categories have less than 5% of prices changing per month, implying average contract lengths of over 40 months. However, the most common contracts durations is one quarter. Table 1's row 2 reports the welfare losses in the BK-GTE under the two policy regimes. Figure 6 plots the optimal weights of each sector in the optimal inflation index.

In Table 1, the conclusion of the previous section appears robust: the welfare gain from switching from the sectoral inflation-targeting rule to the aggregate inflation-targeting rule is as low as 0.006% of steady state consumption. This is the case even though the share of the flexible-contract sector is as high as 30%. The results in the previous sections suggest that the welfare gains between the two inflation-targeting regimes are the largest when the flexible-contract sector has a large share in the economy. In fact, the large share of the flexible-contracts sector in the BK-GTE explains why the welfare gain from moving to sectoral inflation-targeting from aggregate inflation-targeting is larger than in Taylor's US economy. However, the welfare gain from moving to sectoral inflation-targeting is still small due to the presence of a wider range of contracts in the BK-GTE.

Figure 6 here

The implications in the BK-GTE for the optimal weights that sectors receive in the optimal inflation index differ from those found in Taylor's US economy. The distribution of the optimal weights is plotted in Figure 6. The BK-GTE suggests that the longest-contract sector should receive the largest weight in the optimal inflation index and other sectors in which prices are sticky should receive more or less the same weight. The difference in conclusions seems to arise from the fact that, in the BK-GTE, the most common duration is one quarter and a longer duration corresponds to a smaller proportion of firms. In contracts in Taylor's economy, flexible-contracts total only 7%, and the most common durations of $3, 4$ and 5 each have about 20% share in the distribution. Since the most common durations are relatively long and there are only a few firms in the longest-contract sector, the longest contracts (8-period) do not matter much to designing the optimal policy. The

[10]Note that adding more sectors does not change results significantly.

same result also shows how tentative the results are. In fact, the sensitivity of the results to the shape of a distribution makes it difficult to generalize the consequences for optimal weights in a multi-sector economy.

6.3 Discussion

The results above indicate that the improvement in welfare from moving to the sectoral inflation targeting policy is fairly small. Moreover, the sectoral inflation-targeting rule has limitations. The main limitation of this policy is that, as the results in the previous section indicate, it requires measuring all sectoral characteristics. This is a difficult task.

It is also important to note that even though there is more evidence about the degree of nominal rigidity in different sectors than ever before[11], there is still a great deal of uncertainty about the distribution of contract lengths. Take the Bils and Klenow (2004) distribution, for example. In deriving this distribution, following Bils and Klenow (2004), we assume that within each sector there is a Calvo-style contract. However, it is important to recognise that this is just an assumption. The statistic that Bils and Klenow (2004) report is the proportion of firms that change price in a month. This statistic tells us nothing about the distribution of contract lengths within sectors. Thus, this statistic has a limited value and the true distribution may be completely different from what we get when we assume that within each sector there is a Calvo distribution. If this is true, following the sectoral inflation-targeting policy may lead to large welfare losses.

Thus, the central banks may prefer to follow the aggregate inflation targeting policy for two reasons. First, the aggregate inflation targeting policy is easier to implement, as aggregate inflation is readily available. Second, such a rule closely approximates the outcomes under the sectoral inflation targeting policy.

7 Conclusions

When examining the implications of the heterogeneity of contracts lengths on optimal monetary policy design, the standard approach is to consider economies in which there are only two sectors. But more realistic analysis requires considering economies with many sectors, each with a different

[11] See for example Dhyne et al., 2005 and references therein.

contract length and modelling the distribution of contract lengths using empirical data. The purpose of this paper has been to investigate the implications of ignoring the heterogeneity of contract lengths on the optimal choice of inflation index. To accomplish this we have used the Generalized Taylor Economy (GTE) framework to analyze the design of monetary policy rules in an economy where there can be many sectors with different contract lengths. We have generalized the analysis of Rotemberg and Woodford (1998) and have derived a utility-based objective function of a central bank to provide a benchmark for evaluating the performance of alternative inflation-targeting monetary policy rules in an economy in which there are many sectors with different contract lengths. We have then compared the performance of two alternative policy rules. The first is the sectoral inflation-targeting rule under which the nominal interest rate reacts to the appropriately weighted average of the inflation rates in different sectors. The second is the aggregate inflation-targeting rule under which the nominal interest rate targets aggregate inflation. Two results emerge from the analysis in this paper.

First, the results suggest that in a model that assumes an empirically relevant distribution of contract lengths, the performance of an aggregate inflation-targeting scheme closely approximates the performance of a sectoral inflation-targeting rule in which an appropriately weighted average of sectoral inflation rates is targeted. However, a two-sector model suggests that the welfare gain is larger. The main difference between the rules is that aggregate inflation targeting rule requires a larger deviation of output from potential to control price stability. Higher volatility in the output gap when aggregated inflation is targeted leads to a larger deterioration in social welfare in two-sector models than in a model with a distribution. This is because, for a given mean, the presence of long-term contracts in an economy with a wider range of contracts leads to a higher degree of price dispersion. Increased price dispersion makes it more important to control price stability, reducing the importance of the output gap term in the central bank loss function. Therefore, higher volatility in the output gap under the aggregate inflation-targeting scheme is less costly in a multi-sector economy.

Second, if the central bank adopts a sectoral inflation-targeting rule, then the optimal rule does not necessarily place the largest weight on the inflation rate in the sector where the contract length is the longest. In two-sector economies, the optimal weights that sectors receive in the optimal inflation index depend on the contract length in that sector as well as the sector's share. The intuition is simple: if there are only a few firms with long contracts, the behavior of these firms simply does not matter significantly for optimal policy. Although this finding may seem obvious, the existing literature commonly claims that the sector with the longest contract receives the largest weight in the optimal inflation index because of the simplifying assumption that sectors have equal sizes. In a model with a distribution of contract lengths, the optimal weights sectors receive depend on the shape of the distribution. The experiments reported in the paper suggest that if a distribution looks like a geometric distribution (i.e. a longer duration corresponds to a smaller share of the sector), then the sector with the longest contracts receives the greatest weight. On the other hand, if there is a hump-shaped distribution, then the longest-contract sector does not receive the greatest weight. In this case, the sectors in which prices are sticky but have a large weight in the sectoral index receive a larger weight than the longest-contract sector. This sensitivity makes it difficult to generalize results of optimal weights when there is a distribution of contract lengths.

The results suggest that central banks may still prefer to follow the aggregate inflation-targeting rule rather than the sectoral inflation targeting rule. Such a rule is easier to implement, since this rule does not require to measuring all sectoral characteristics and since it closely approximates the performance of the sectoral inflation-targeting.

8 Appendix: Derivation of the welfare function

A second-order approximation of the period utility $U(C_t)$ around steady state yields:

$$U_t(C) = U_C(C)C(c_t + \frac{1-\eta_{cc}}{2}c_t^2) + t.i.p + O(\|a\|^3), \tag{26}$$

where c_t denotes the log-deviation of consumption from steady state, $t.i.p$ collects all the terms that are independent of policy and $O(\|a\|^3)$ summarizes all terms of the third or higher orders.

Using the fact that $\tilde{c} = \tilde{y}_t$ in our model and the definition $\tilde{y}_t = y_t - y_t^*$, (26) can be expressed in terms of the output gap

$$U_t(C) = U_C C \left(\tilde{y}_t + \frac{1-\eta_{cc}}{2}\tilde{y}_t^2 + (1-\eta_{cc})\tilde{y}_t y_t^* \right) + +t.i.p + O(\|a\|)^3$$

Similarly, taking a second order approximation of $V(1-L_t)$ around steady state and using the definition $\widetilde{l_t} = l_t - l_t^*$ yields

$$V\left(1-L_t\right) = -V_L(1-L_t)L\left(\tilde{l}_t + \frac{(1+\eta_{ll})}{2}\tilde{l}_t^2 + (1+\eta_{ll})\tilde{l}_t l_t^* \right) + t.i.p + O(\|a\|)^3 \tag{27}$$

Using (6) and aggregating for cohort j in sector i gives

$$\tilde{l}_t = \tilde{y}_t + \tilde{u}_t \tag{28}$$

where u_t is given by

$$\tilde{u}_t = \sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\left(\frac{P_{ijt}}{P_t}\right)^{-\theta} - \sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\left(\frac{P_{ijt}^*}{P_t^*}\right)^{-\theta} \tag{29}$$

A second order approximation of $\tilde{u}_t$ yields

$$\tilde{u}_t = \frac{1}{2}\theta\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\tilde{p}_{ijt}^2 + O(\|a\|)^3 \tag{30}$$

Proof.

$$\left(\frac{P_{ijt}}{P_t}\right)^{-\theta} - \left(\frac{P^*_{ijt}}{P^*_t}\right)^{-\theta} = e^{-\theta\tilde{p}_{ft}}$$

where $\tilde{p}_{ft} = \bar{p}_{ft} - \bar{p}^*_{ft}$. Taking a second-order approximation around the steady state and noting that all firms in the same cohort face the same wage and hence set the same price $p_{ft} = p_{ijt}$, one obtains

$$e^{-\theta\tilde{p}_{ijt}} = 1 - \theta\tilde{p}_{ijt} + \frac{1}{2}\theta^2\tilde{p}^2_{ijt}$$

Summing over the cohorts in sector i and then summing over the sectors yield

$$\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}e^{-\theta\tilde{p}_{ijt}} = \{1 - \theta\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\tilde{p}_{ijt} \ + \frac{1}{2}\theta^2\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\tilde{p}^2_{ijt}\} \quad (31)$$

Using the definition of price P_t and taking a second order approximation of $e^{(1-\theta)\tilde{p}_{ijt}}$,we obtain

$$\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\tilde{p}_{ijt} = -\frac{1}{2}(1-\theta)\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\tilde{p}^2_{ijt} \quad (32)$$

Combining (31) and (32), we obtain

$$\tilde{u}_t = 1 + \frac{1}{2}\theta\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\tilde{p}^2_{ijt} \quad (33)$$

■

Plugging (30), (28) into (27), summing the resulting equation with (26), and using the steady-state relations $U_C(C)C = V_L(1-L)L$ and $(1-\eta_{cc})y^*_t = (1+\eta_{ll})l^*_t$, we obtain

$$U_t(C)+V(1-L_t) = -\frac{U_C(C)C}{2}\left((\eta_{cc}+\eta_{ll})\tilde{y}^2_t + \theta\sum_{i=1}^{N}\sum_{j=1}^{T_i}\alpha_i\lambda_{ij}\tilde{p}^2_{ijt}\right)+t.i.p+O(\|a\|)^3 \quad (34)$$

References

Aoki, K., 2001. Optimal monetary policy response to relative price changes. Journal of Monetary Economics 48, 55–80.

Ascari, G., 2000. Optimising agents, staggered wages and the persistence in the real effects of money shocks. The Economic Journal 110, 664–686.

Benigno, P., 2004. Optimal monetary policy in a currency area. Journal of International Economics 63, 293–320.

Bils, M., Klenow, P., 2004. Some evidence on the importance of sticky prices. Journal of Political Economy 112, 947–985.

Blanchard, O., Gali, J., 2007. Real wage rigidities and the New Keynesian model. Journal of Money, Credit and Banking 39, 35–65.

Carvalho, C., 2006. Heterogeneity in price stickiness and the real effects of monetary shocks. Frontiers of Macroeconomics 2.

Dhyne, E., Alvarez, L. J., Bihan, H. L., Veronese, G., Dias, D., Hoffmann, J.,Jonker, N., Lunnemann, P., Rumler, F. and Vilmunen, J., 2005. Price setting in the euro area: some stylized facts from individual consumer price data. European Central Bank Working paper series 524.

Dixon, H., Kara, E., 2006. How to compare Taylor and Calvo contracts: a comment on Michael Kiley. Journal of Money, Credit and Banking 38, 1119-1126.

Dixon, H., Kara, E., 2007. Persistence and nominal inertia in a Generalized Taylor Economy: how longer contracts dominate shorter contracts? Cardiff Business School Working paper series 2007/1.

Dixon, H., Kara, E., 2010. Can we explain inflation persistence in a way that is consistent with the microevidence on nominal rigidity. Journal of Money, Credit and Banking 42, 151 - 170.

Edge, R., 2002. The equivalence of wage and price staggering in monetary business cycle models. Review of Economic Dynamics 5, 559–585.

Erceg, C. J., Henderson, D., Levin, A., 2000. Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics 46, 281–313.

Erceg, C. J., Levin, A., 2006. Optimal monetary policy with durable consumption goods. Journal of Monetary Economics 53, 1341–1359.

Gali, J., 2003. New Perspectives on Monetary Policy, Inflation, and the Business Cycle, in Advances in Economic Theory, in Dewatripont M., Hansen L., Turnovsky S. (Eds.), Vol. III, Cambridge University Press, pp. 151-197.

Huang, K. X. , Liu, Z., 2005. Inflation targeting: What inflation rate to target? Journal of Monetary Economics 52, 1435–1462.

Juillard, M., 1996. Dynare: a program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm, CEPREMAP working paper no. 9602.

Mash, R., 2004. Optimising microfoundations for inflation persistence. Oxford University discussion paper 183.

Paustian, M., 2005. The role of contracting schemes for the welfare costs of nominal rigidities over the business cycle. Deutsche Bundesbank discussion papers Series No. 22/2005 .

Rotemberg, J. J., Woodford, M., 1998. An optimization-based econometric framework for the evaluation of monetary policy: Expanded version. NBER Working Paper No. t0233 .

Sheedy, K., 2007. Intrinsic inflation persistence. CEP discussion paper no: 837.

Taylor, J. B., 1993. Macroeconomic Policy in a World Economy: From Econometric Design to Practical Operation. W W Norton.

Wolman, A. L., 1999. Sticky prices, marginal cost, and the behavior of inflation. Federal Reserve Bank of Richmond Economic Quarterly,85, 29– 48.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton Univeristy Press, Princeton, NJ.

	sectoral	aggregate
	inflation-targeting rule	inflation-targeting rule
Taylor's US Economy	0.001	0.003
BK-GTE	0.002	0.008

Table 1: The welfare losses in terms of (%) change in steady-state consumption relative to the optimal policy

	sectoral	aggregate
	inflation-targeting rule	inflation-targeting rule
Taylor's US Economy	0.15	0.30
BK-GTE	0.23	0.51

Table 2: Standard deviation of the output gap (%)

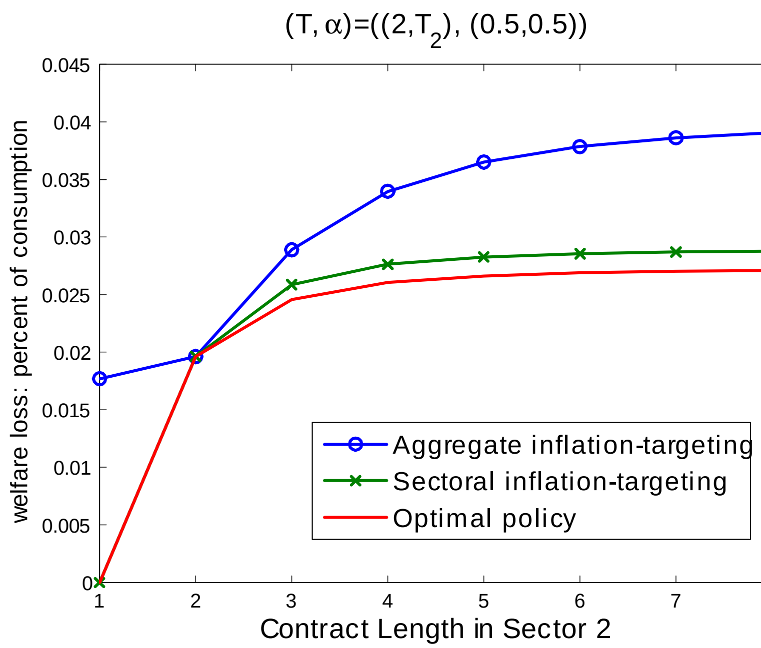

Figure 1: Welfare comparisons in terms of the equivalent % decline in steady state consumption.

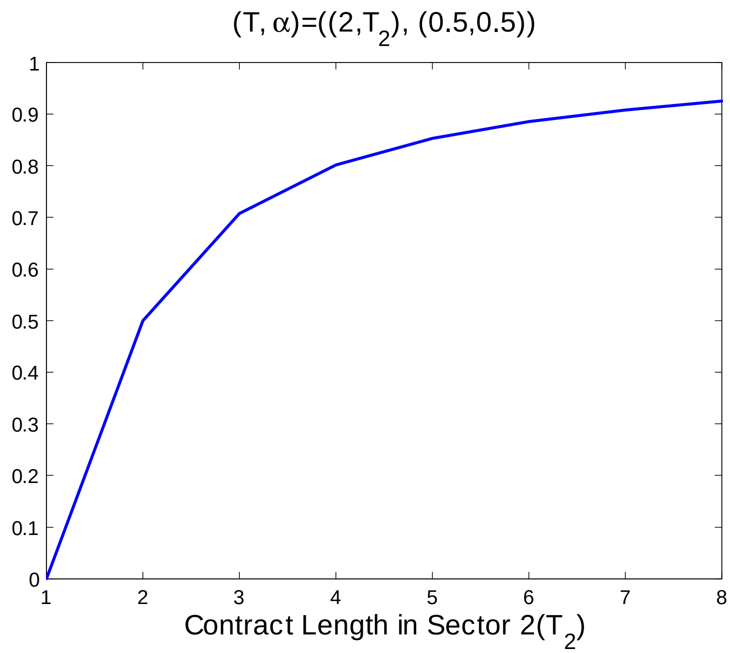

Figure 2: The weight on Sector 2 in the optimal index

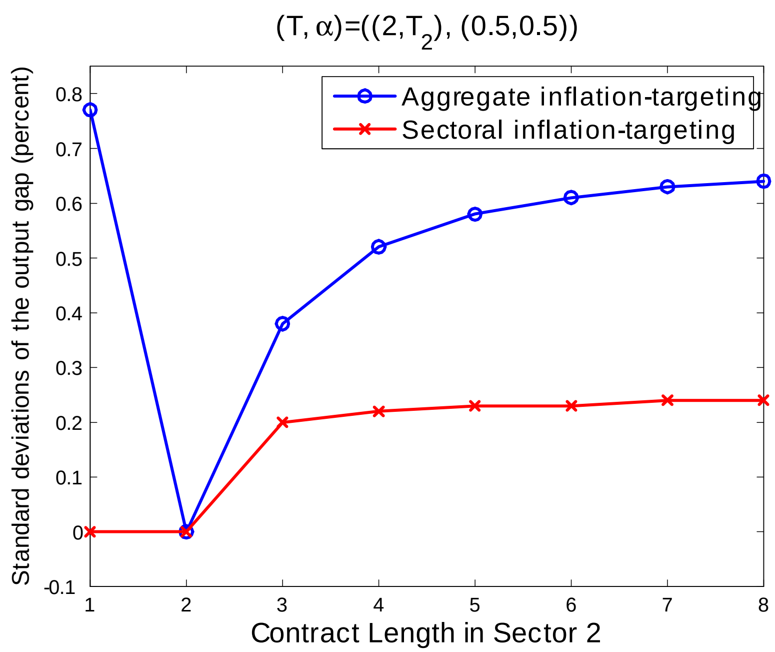

Figure 3: Standard deviation of output gap under different policies

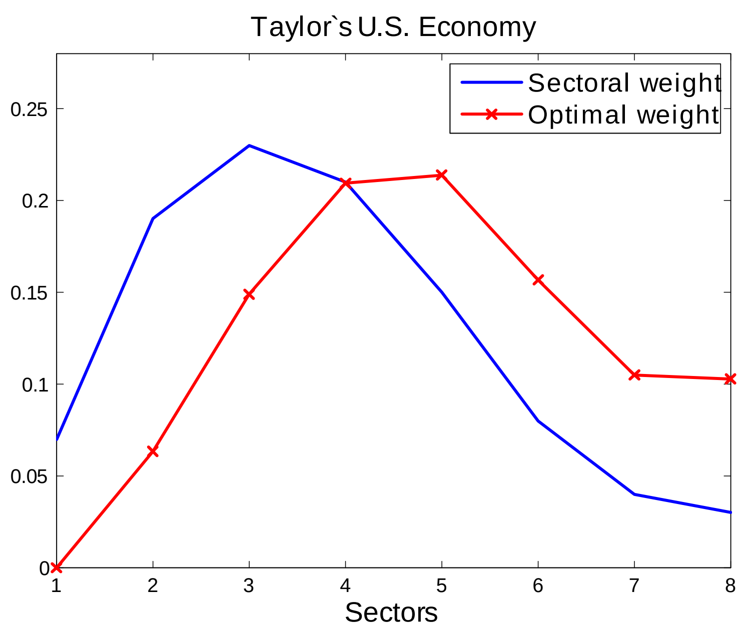

Figure 4: The weight on different sectors in Taylor's US Economy

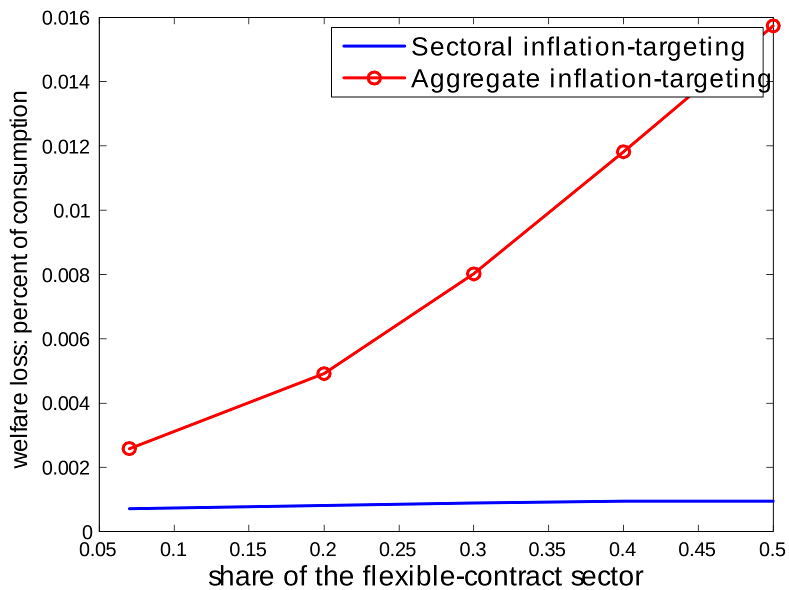

Figure 5: The welfare losses in terms of % change in steady-state consumption relative to the optimal policy

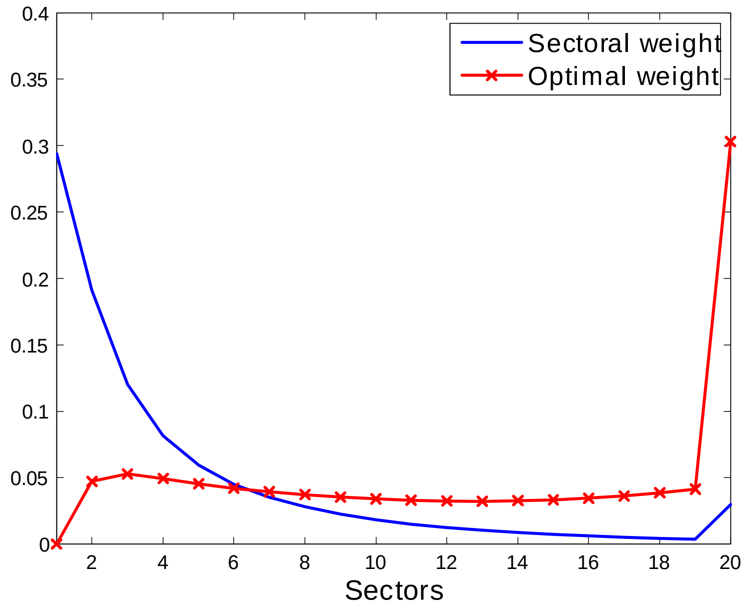

Figure 6: The weights on different sectors in the BK-GTE