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Abstract

This paper characterizes the optimal policy within a dynamic search model of the labor market with risk-averse workers. In a first-best allocation of resources, unemployment benefits should provide perfect insurance against the unemployment risk, layoff taxes are necessary to induce employers to internalize the cost of dismissing an employee but should not be too high in order to allow a desirable reallocation of workers from low to high productivity jobs, hiring subsidies are needed to partially offset the adverse impact of layoff taxes on job creation and payroll taxes should be approximately equal to zero. I derive an *optimal rate of unemployment* and show that it is lower when the unemployment risk is partly non-insurable. Importantly, the optimal level of layoff taxes and hiring subsidies is independent of the amount of government expenditures to finance, even in a second-best environment. When workers have some bargaining power, it is optimal to reduce the rate of job creation below the output maximizing level in order to lower wages and increase the level of unemployment benefits. Thus, layoff taxes should typically exceed hiring subsidies which generates enough surpluses to finance at least some of the unemployment benefits. The inclusion of moral hazard does not change this conclusion, unless workers have very low bargaining power.

Keywords: Unemployment insurance, Employment protection, Hiring subsidies, Optimal rate of unemployment

JEL Classification: D60, E62, H21, J38, J65

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1 Introduction

The design of labor market institutions is widely believed to be among the key determinants of the economic success or failure of a nation. There is nevertheless no consensus among economists about the optimal design of such institutions and, in many industrialized countries, the subject remains at the center of considerable controversies among policy makers. In particular, there appear to be a fundamental trade-off between the demand for insurance of risk-averse workers and the macroeconomic efficiency of the labor market which should allocate workers to the jobs where they are going to be most productive. Hence, a typical concern is that government interventions aimed at improving insurance, such as the provision of unemployment benefits or employment protection, might also have adverse consequences for aggregate production.

Search frictions are a major source of the trade-off between insurance and production[1] since they generate some unemployment and they prevent an immediate reallocation of workers from low to high productivity jobs. A macroeconomic framework is required to analyze this trade-off as search frictions induce non-trivial general equilibrium effects on job creation and job destruction which are key to the reallocation process of workers. Furthermore, wages could be affected by macroeconomic variables such as the expected length of an unemployment spell. These general equilibrium effects imply that different labor market policy instruments do interact among each other. They therefore jointly influence the provision of insurance and the efficiency of production.

A search model *à la* Mortensen-Pissarides (1994) with risk-averse workers captures all the above features and allows for a joint analysis of the different policy instruments. In this paper, I therefore rely on such a framework to determine the main characteristics of an optimal labor market policy. Employment protection takes the form of layoff taxes. The government can also give hiring subsidies to encourage job creation. The generosity of unemployment insurance is determined by the level of unemployment benefits. Payroll taxes could be used to raise revenue. If they happen to take negative values, payroll taxes could also be seen as employment subsidies. Importantly, it is assumed throughout, as in most of the literature on the topic, that the government is the sole provider of unemployment insurance.[2]

I begin by deriving the optimal allocation of resources chosen by a planner who wants

[1] The other major source of the trade-off is moral hazard which will be allowed towards the end of this paper.

[2] The implicit contract literature has argued that risk-neutral firms should be expected to provide unemployment benefits to risk-averse workers; see, for instance, Baily (1974a) or Azariadis (1975). However, in reality, such contracts remain the exception rather than the rule. Thus, although somewhat *ad-hoc*, the assumption that the private market does not provide insurance seems reasonable and has the merit of making the analysis transparent. This assumption has nevertheless been relaxed in the optimal policy analyses of Fella (2007) and Chetty Saez (2008).

to maximize the welfare of workers subject to matching frictions and to a resource constraint. In this ideal setup, full insurance is provided and aggregate output, net of recruitment costs, is maximized. It turns out that this first-best allocation could be implemented in a decentralized economy when workers are wage takers. To obtain an efficient rate of job destruction, layoff taxes should induce firms to internalize the social costs and benefits of dismissing a worker. The costs consist of the unemployment benefits that will need to be paid and of the forgone payroll taxes; while the benefit corresponds to the value of a desirable reallocation of the worker from a low to a high productivity job. Hiring subsidies are needed to partially offset the negative impact of layoff taxes on job creation. Finally, and perhaps surprisingly, payroll taxes should optimally be approximately equal to zero. Thus, both unemployment benefits and hiring subsidies are almost entirely financed from layoff taxes.

I then consider a number of deviations from this first-best benchmark. First, I show that additional government expenditures, to provide public goods for instance, should be *exclusively* financed through higher payroll taxes and lower unemployment benefits, even if this induces a downward distortion to the participation decision of workers. Layoff taxes should therefore be seen as a Pigouvian instrument which corrects for inefficiencies in the rate of job destruction, not as a source of revenue to the government. I then turn to the possibility of a non-insurable utility cost of unemployment. In this context, it is optimal to reduce the rate of unemployment, which acts as a substitute to the provision of insurance through unemployment benefits. However, the lower rate of unemployment slows down the reallocation of workers and therefore fails to maximize output. This illustrates the conceptual distinction between the welfare maximizing *optimal rate of unemployment*[3] derived in this paper and the output maximizing rate of unemployment which is central to the search-matching literature.

I then rely on numerical simulations to explore the optimal policy when workers have some bargaining power. As the provision of insurance tends to be insufficient, the planner wants to reduce market tightness in order to decrease wages which, by relaxing the resource constraint, allows an increase in the level of unemployment benefits. This is achieved by setting layoff taxes higher than hiring subsidies in order to discourage the entry of firms with a vacant position. I then allow for moral hazard which generates the opposite possibility that insurance may be too *high*, in which case the planner wants to *increase* market tightness. However, the simulations reveal that under-insurance remains the main concern whenever workers have substantial bargaining power. Thus, moral hazard does not seem to be the most important feature of the fundamental trade-off between the provision of insurance and the level of aggregate production. General equi-

[3] To the best of my knowledge, there is no other paper which derives such an optimal rate of unemployment properly microfounded in terms of the individual risk-averse preferences of workers.

librium effects on wages and on job creation and job destruction seem to be at least as important.

This paper is related to the extensive economic literature on optimal labor market institutions. The main strand of this literature is on optimal unemployment insurance. In their seminal work, Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) focused on a single unemployment spell and derived the optimal time profile of unemployment benefits when moral hazard introduces a trade-off between the provision of insurance and incentives to search. By contrast, Baily (1974b) and Chetty (2006) focused on the level of benefits, rather than their time profile, in a framework which allows for multiple spells. Importantly, these contributions assume that unemployment benefits are exclusively financed from payroll taxes and abstract from general equilibrium effects.

The literature on employment protection is mostly positive, rather than normative. The crux of the academic debate is about the impact of layoff taxes on the level of employment; with the underlying presumption that layoff taxes are desirable if they decrease the number of jobless. Bentolila and Bertola (1990) showed, in a partial equilibrium context, that firing costs have a larger impact on job destruction than on job creation and should therefore be beneficial for employment. This conclusion was challenged by the general equilibrium analysis with employment lotteries of Hopenhayn and Rogerson (1993). Ljungqvist (2002) showed that, in search models *à la* Mortensen-Pissarides, layoff costs increase employment if initial wages are negotiated before a match is formed, while the opposite is true if bargaining only occurs after the match is formed. Importantly, these contributions either assume that workers are risk-neutral or that financial markets are complete. Hence, they do not generate any trade-off between insurance and production efficiency and cannot give sensible measures of the welfare implications of layoff taxes. These analyses are therefore hardly informative about the optimal level of employment protection.

While most papers ignore the interaction between different policy instruments, there are two important exceptions which are closely related to this work. First, Mortensen and Pissarides (2003)[4] analyze labor market policies in a dynamic search model with risk-neutral workers. Since there is no motive for insurance, the best that the government can do is to maximize output net of recruitment costs. If the Hosios (1990) condition holds, i.e. the bargaining power of workers is equal to the elasticity of the matching function, then it is optimal for the government not to intervene. While, if it does not hold, policy parameters should only be used to correct for the resulting search externalities. An important insight is that the introduction of unemployment benefits has a positive impact on wages and, therefore, increases job destruction. This should be offset by higher layoff taxes. Hiring subsidies should also be increased such as to leave the rate of job

[4]See also Mortensen Pissarides (1999) and Pissarides (2000, chapter 9).

creation unchanged. However, with risk-neutral workers, there is no trade-off between insurance and production.

The second closely related paper is Blanchard Tirole (2008) which proposes a joint derivation of optimal unemployment insurance and employment protection in a static context with risk-averse workers. They show in a benchmark model, which is the static counterpart to the first-best policy derived in this paper, that unemployment benefits should be entirely financed from layoff taxes, rather than payroll taxes, in order to induce firms to internalize the cost of unemployment.[5] However, their static framework ignores the adverse effect of layoff taxes on job creation. In fact, as I shall show, in a dynamic context the share of unemployment benefits financed from payroll taxes is determined by the job creation side of the economy, which is absent from their framework. Also, and more fundamentally, a static approach entails an entirely negative view of unemployment; whereas in a dynamic setting an unemployed worker is a useful input in the matching process. In fact, to maximize output in an economy without governmental intervention, the Hosios condition actually *maximizes* the rate of job destruction!

Finally, this paper is also related to a small literature on policy analyses within dynamic search models of the labor market with risk-averse workers. Cahuc Lehmann (2000), Fredriksson Holmlund (2001) and Lehmann van der Linden (2007) focus on the optimal provision of unemployment insurance under moral hazard. All three contributions pay particular attention to the general equilibrium effects of unemployment insurance and to their consequences for the overall provision of insurance. Interactions with layoff taxes are nevertheless ignored.

Acemoglu Shimer (1999, 2000) showed, in the context of directed search with risk-averse workers, that higher unemployment benefits could improve the quality, and productivity, of job-worker matches. By contrast, in this paper, match quality is unrelated to the length of unemployment. Alavarez Veracierto (2000, 2001) rely on calibrated search models with risk-averse workers to investigate the effects of different labor market policies. However, their approach is entirely positive and does not attempt to characterize optimal policies.[6]

In a closely related paper, Coles and Masters (2006) show that there is some complementarity between the provision of unemployment insurance and that of hiring subsidies. The idea is that, by boosting the job creation rate, subsidies exert a downward pressure on unemployment and, hence, on the cost of providing unemployment insurance. How-

[5] This policy, often referred to as "experience rating", was originally proposed by Feldstein (1976). Other related contributions on the topic, and mostly in favor of such policy, include Topel Welch (1980), Topel (1983), Wang Williamson (2002), Cahuc Malherbet (2004), Mongrain Roberts (2005), Cahuc Zylberberg (2008) and L'Haridon Malherbet (2009).

[6] Ljungqvist Sargent (2008) also investigate the interactions between unemployment insurance and employment protection in a positive analysis of the labor market, but with risk-neutral workers.

ever, their model does not have an endogenous job destruction margin and, therefore, cannot be used to determine the optimal level of employment protection.

This paper begins, in section two, with a brief reminder of the key features of the Mortensen-Pissarides (1994) framework, on which all subsequent work relies. In the following section, I derive the first-best policy, which then serves as a benchmark. Section four investigates how government expenditures should be financed when payroll taxes and layoff taxes are both potential sources of revenue. I then turn to the consequences of a non-insurable utility cost of unemployment. Section six relies on numerical simulations to investigate optimal policies when workers have some bargaining power. Finally, the last section deals with the consequences of moral hazard. This paper ends with a conclusion.

2 Search Model

Before solving for optimal policies, it is necessary to describe the main characteristics of the dynamic search model on which all subsequent work relies. The structure of the economy corresponds to the standard Mortensen-Pissarides (1994) framework. Production requires that vacant jobs and unemployed workers get matched, which occur at rate:

$$m = m(u, v), \tag{1}$$

where u stands for the number of unemployed and v for that of vacancies. For simplicity, each firm can employ, at most, one worker and the mass of workers is normalized to one, so that u also stands for the rate of unemployment. The matching function m is increasing in both arguments, exhibits decreasing marginal product to each input and satisfies constant returns to scale. It follows from this last assumption that the key parameter of interest, which summarizes labor market conditions, is market tightness defined as the ratio of vacancies to unemployment, $\theta = v/u$. The rate at which vacant jobs meet unemployed workers is given by:

$$\frac{m(u,v)}{v} = m\left(\frac{u}{v}, 1\right) = m\left(\frac{1}{\theta}, 1\right) = q(\theta), \tag{2}$$

where q is a decreasing function of θ. Similarly the rate at which unemployed workers find jobs is:

$$\frac{m(u,v)}{u} = m(1, \theta) = \theta q(\theta). \tag{3}$$

The elasticity of the matching function, to which I will subsequently refer, is defined as:[7]

$$\eta(\theta) = -\frac{\theta}{q(\theta)}\frac{dq(\theta)}{d\theta}. \tag{4}$$

The other main feature of the Mortensen-Pissarides model is that the productivity of a match is subject to idiosyncratic shocks. Production starts at maximal productivity, normalized to 1. The idea is that recruiting firms are prosperous and initially provide their employees with the best available technology.[8] At Poisson rate λ, the match is hit and a new productivity $x \in [\psi, 1]$ is randomly drawn from c.d.f. $G(x)$. The match dissolves if the new productivity is below a threshold R, to be determined. Additional details will be given as the optimal policy is being derived.

3 First-Best Policy

The optimal policy is derived in two steps. First, I characterize the optimal allocation of resources chosen by a benevolent social planner. Then, I turn to its implementation in a decentralized economy with free entry of risk-neutral firms.

3.1 Optimal Allocation

The optimal allocation maximizes a utilitarian social welfare function subject to a resource constraint and to the search frictions that characterize the labor market. It is therefore the solution to the following problem:

$$\max_{\{\theta,R,b,w\}} \int_0^\infty e^{-\rho t}\left[(1-u)v(w) + uv(z+b)\right] dt \tag{5}$$

$$\text{subject to} \quad \dot{u} = \lambda G(R)(1-u) - \theta q(\theta) u \tag{6a}$$

$$\dot{y} = \theta q(\theta) u + \lambda(1-u)\int_R^1 s dG(s) - \lambda y \tag{6b}$$

$$(1-u)w + ub = y - c\theta u \tag{6c}$$

where ρ stands for the planner's (or workers') discount rate, w for the net wage that an employee receives, z for the value of leisure, b for unemployment benefits, y for the aggregate output of the economy and c for the flow cost of posting a vacancy. The in-

[7] Note that η is the elasticity of the matching function with respect to the number of unemployed, i.e. $\eta = \frac{u}{m}\frac{\partial m}{\partial u}$, and $1-\eta$ the elasticity with respect to the number of vacancies, i.e. $1-\eta = \frac{v}{m}\frac{\partial m}{\partial v}$.

[8] This assumption, which is standard in the search-matching literature, is also made for convenience and its importance should not be overstated. Indeed, firms base their recruiting decisions on the expected net present value of a new match rather than on its initial productivity.

stantaneous utility function of risk-averse workers is denoted by[9] $v(.)$, which is increasing and concave.

The planner's objective is to maximize intertemporal social welfare, which, following a utilitarian criteria, is composed, at each instant, of the instantaneous utility of u unemployed and $1-u$ employed workers[10]. The first constraint depicts the dynamics of unemployment, driven by the difference between the job destruction flow and the job creation flow. A match dissolves when it is hit by an idiosyncratic shock that generates a new productivity below the threshold R, which occurs at rate $\lambda G(R)$. This rate of job destruction applies to the mass $1-u$ of existing matches. Job creation is simply equal to the rate at which unemployed workers find jobs, $\theta q(\theta)$, multiplied by the mass u of job seekers. It should be emphasized that this first constraint captures the fact that even the social planner is subject to matching frictions. The second constraint gives the dynamics of aggregate output, y. At each instant, $\theta q(\theta)u$ new matches are formed and each of these has a productivity of 1. The $1-u$ existing jobs are hit at rate λ by idiosyncratic shocks which destroy their current productivity and replaces it, in case of survival, by a randomly drawn number greater or equal to the threshold R. Finally, any feasible allocation must satisfy the economy's resource constraint. The expenses, composed of the wages paid to the employed and the benefits paid to the unemployed, cannot exceed total output net of the resources allocated to recruitment, which amount to a flow cost c paid for each of the θu vacancies. The planner's control variables are market tightness θ, threshold productivity R, net wage w and unemployment benefits b. The state variables are unemployment u and aggregate output y.

The planner's problem is straightforward to solve using standard optimal control techniques. The first characteristic of the optimal allocation is perfect insurance for workers:

$$w = z + b, \tag{7}$$

which follows directly from risk aversion, i.e. from the concavity of $v(.)$. This could be combined with the resource constraint, (6c), to give the optimal value of w and b:

$$w = y - c\theta u + zu, \tag{8}$$

$$b = y - c\theta u - z(1-u). \tag{9}$$

[9]In the previous section v denoted the number of vacancies. However, this variable will not appear in the rest of the text (except when I define the matching function under moral hazard in the last section of the paper). I focus instead on θ and u and, where needed, v is just replaced by θu.

[10]An alternative would be to maximize the weighted average between the expected utility of an employed and of an unemployed worker. Such objective function would be more appropriate for political economy work focusing on the conflict between insiders and outsiders. However, without time discounting, this would be identical to the planner's objective retained in this paper.

Note that perfect insurance necessitates a replacement ratio smaller than one whenever the value of leisure, z, is strictly positive. The optimal value of θ and R is implicitly determined by the following two first-order conditions:

$$[1-\eta(\theta)]\,\frac{1-R}{\rho+\lambda}=\frac{c}{q(\theta)}, \tag{10}$$

$$R=z+\frac{\eta(\theta)}{1-\eta(\theta)}c\theta-\frac{\lambda}{\rho+\lambda}\int_R^1(s-R)dG(s), \tag{11}$$

where $\eta(\theta)$ denotes the elasticity of the matching function, cf. equation (4). These two optimality conditions are exactly identical to the one derived in Pissarides (2000, chapter 8) for net[11] output maximization. This is not surprising as, when nothing prevents the provision of full insurance, the best that the planner can do is to maximize output. The first equation, (10), corresponds to optimal job creation. The cost of job creation consists of the flow cost of having a vacancy, c, multiplied by the expected time that has to be spent before a worker could be found, $1/q(\theta)$. The value of a newly created match is equal to $(1-R)/(\rho+\lambda)$. However, optimally, recruitment costs should only absorb a fraction $1-\eta(\theta)$ of this value, otherwise there is too much job creation and an excessive amount of resources is allocated to recruitment. Equation (11) gives optimal job destruction. In the static context of Blanchard Tirole (2008), the optimal threshold is just equal to the value of leisure, i.e. $R=z$. Making the model dynamic yields two extra terms. First, when a low productivity job is destroyed, the corresponding worker returns to unemployment with the hope of finding a new job with productivity 1. To make this explicit, the corresponding term of equation (11) could be rewritten, using (10), as:

$$\begin{aligned}\frac{\eta(\theta)}{1-\eta(\theta)}c\theta &= \theta q(\theta)\eta(\theta)\frac{1-R}{\rho+\lambda}\\ &= \theta q(\theta)\left[\frac{1-R}{\rho+\lambda}-\frac{c}{q(\theta)}\right].\end{aligned} \tag{12}$$

This says that, once a job is destroyed, an unemployed worker gets matched at rate $\theta q(\theta)$ which generates a social value of $(1-R)/(\rho+\lambda)$ net of the expected recruitment cost $c/q(\theta)$. In other words, the threshold R has to be sufficiently high to induce an efficient reallocation of workers from low to high productivity jobs. The second additional term to the expression for the optimal threshold R corresponds to the option value of a match. Even if current productivity is very low, keeping the match alive preserves the option of being hit by an idiosyncratic shock that restores a profitable level of productivity. The option value decreases the optimal threshold R.

The optimal allocation of resources chosen, in steady state, by a benevolent social

[11] Under risk neutrality, the optimal policy is to maximize the net present value of the flow of net output, where this flow is given by $y-c\theta u+uz$.

planner is characterized by the first-order conditions (7), (10) and (11) together with the constraints (6a), (6b) and (6c) with $\dot{u} = \dot{y} = 0$.

3.2 Implementation

Having characterized the optimal allocation, I now turn to its implementation in a decentralized economy. Four stages of interest could be distinguished.

- Stage 1: The government chooses the level of unemployment benefits b, payroll taxes τ, layoff taxes F and hiring subsidies H.
- Stage 2: Entrepreneurs decide whether or not to create a firm with a vacant position.
- Stage 3: Once a match occurs, the employer and employee agree on a wage rate.
- Stage 4: Firms choose a threshold productivity R below which a match hit by an idiosyncratic shock dissolves.

I now proceed by backward induction and start by determining the threshold R chosen by a risk-neutral employer. The asset value of a producing firm with productivity x, $J(x)$, solves the following Bellman equation:

$$rJ(x) = x - (w + \tau) + \lambda \int_R^1 J(s)dG(s) - \lambda G(R)F - \lambda J(x), \tag{13}$$

where r denotes the interest rate, w the net wage that the worker receives and $w + \tau$ the gross wage paid by the employer. Note that, in this framework, the planner's discount rate ρ does not have to coincide with the economy's interest rate r. This Bellman equation states that, for a firm, the flow return from having a filled job with productivity x is equal to the instantaneous surplus it generates to which the possibility of a change in productivity should be added. An idiosyncratic shock destroys the value of the firm at the current productivity and replaces it by either a corresponding expression, if the new productivity is above the threshold, or by the cost of layoff[12], if the match is to be destroyed. As $J(x)$ is strictly increasing in x, employers' chosen threshold R is determined by:

$$J(R) = -F. \tag{14}$$

[12] Throughout this paper, it is assumed that firms are able to pay the layoff tax. Blanchard and Tirole (2008) investigate the consequences of having employers constrained by shallow pockets. See also Tirole (2009) for a deeper analysis on the topic which allows for extended liability to third parties.

This says that, at the threshold, employers are indifferent between closing down and continuing the relationship. Simple algebra[13] on (13) and (14) gives the expression for the value of R chosen by firms:

$$R = w + \tau - rF - \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s). \tag{15}$$

The threshold productivity is smaller than the cost of labor because of the firing tax and of the option value of continuing the match. Note that, for this to be possible, firms must be able to borrow and lend from perfect financial markets, an assumption that is maintained throughout this paper. Equation (15) is our first implementability constraint.

Let us now turn to the determination of the wage rate that occurs at Stage 3. The formation of a match generates a surplus that needs to be shared between the two parties. But, from equation (7), optimality requires that the net wage paid to a worker, w, is equal to the wage equivalent of being unemployed, $z + b$. This leads to following lemma:

Lemma 1 *A necessary condition to implement the first-best allocation is that workers are wage takers and that all the surplus from matches is captured by firms. This ensures that, as desired:*

$$w = z + b. \tag{16}$$

The intuition for this result is straightforward. If workers have some bargaining power, they will obtain a mark-up over and above their outside option which is the income they get while unemployed. But this prevents the provision of full insurance which is a characteristic of a first-best allocation.[14] Clearly, with a binding resource constraint (6c) and perfect insurance, the optimal values of w and b are still given by (8) and (9), respectively.

In the context of this paper, the requirement that workers have no bargaining power could also be seen as part of the optimal policy to be implemented[15]. For example, the labor market could be organized in such a way that firms and workers first meet without exchanging any information on the wage rate. Then, firms make a take-it-or-leave-it offer to workers. Note that, here, a minimum wage would be detrimental to insurance. Excessive monopsony power of firms should rather be dealt with traditional

[13] An analytic expression for the function $J(.)$ could be obtained by taking the difference between equation (13) evaluated at x and the same equation evaluated at R. This expression for $J(.)$ could then be substituted into (13) evaluated at R. Finally, (15) is obtained by plugging (14) in.

[14] It should be noted that, in their benchmark case, Blanchard and Tirole (2008) also assume that the bargaining power of workers is nil. Thus, the first-best benchmark derived in this section is a dynamic counterpart to theirs.

[15] In an environment with Nash bargaining, one solution proposed by Lehmann and van der Linden (2007) consists in setting a marginal rate of income taxation equal to 100%.

policy instruments such as payroll and layoff taxes, hiring subsidies and unemployment benefits.[16]

Finally, the following corollary is an immediate consequence of the above lemma:

Corollary 1 *The first-best allocation cannot be implemented when the Hosios condition holds, i.e. when the bargaining power of workers is equal the elasticity of the matching function* $\eta(\theta)$*.*

The Hosios condition balances search externalities on both sides of the market such that, without government intervention, output is maximized. It is, however, inconsistent with the provision of perfect insurance. Since the optimal allocation of resources is characterized by output maximization, cf. (10) and (11), and workers have zero bargaining power, the optimal policy will correct the rates of job creation and job destruction for the failure of the Hosios condition to hold.

Stage 2 is solved by assuming free entry. Vacancies keep being created by entrepreneurs until the returns from doing so reduce to zero. More formally, the value of a vacant position, V, solves:

$$rV = -c + q(\theta)\left[J(1) + H - V\right]. \tag{17}$$

This states that the return from a vacancy consists of the flow cost of recruitment, c, and of the possibility of filling the position at rate $q(\theta)$ which yields the value of an active firm with productivity 1. The employer also qualifies for a hiring subsidy, H, when he hires a worker. Free entry implies:

$$V = 0. \tag{18}$$

The amount of job creation could then be determined by plugging (18) into (17) and by using the value of $J(1)$ deduced from (13) and (14). This gives:

$$\frac{1-R}{r+\lambda} - F = \frac{c}{q(\theta)} - H. \tag{19}$$

The left hand side is the value of a new match to a firm, $J(1)$; while the right hand side corresponds to the expected cost of recruiting a worker. Equation (19) is our second implementability condition.

At Stage 1, the government needs to choose the optimal policy. The corresponding implementability condition is the usual government budget constraint:

$$(1-u)\tau + (1-u)\lambda G(R)F = ub + u\theta q(\theta)H. \tag{20}$$

Revenues consist of payroll taxes paid by employed workers and of layoff taxes applied to the job destruction flow; while the expenses are the payment of benefits to the unemployed

[16] See Cahuc Laroque (2009) for a similar argument in a redistributive context.

and of hiring subsidies to the flow of newly created jobs.

It is now straightforward to find the optimal policy by matching the implementability conditions to the equations that characterize the first-best allocation. More specifically, (19) should be combined with (10) and (15) with (11). This gives:

$$F - H = \eta(\theta)\frac{1-R}{\rho+\lambda} + \frac{\rho - r}{r+\lambda}\frac{1-R}{\rho+\lambda}, \tag{21}$$

$$rF = b + \tau - \frac{\eta(\theta)}{1-\eta(\theta)}c\theta + \frac{r-\rho}{r+\lambda}\frac{\lambda}{\rho+\lambda}\int_R^1 (s-R)dG(s), \tag{22}$$

where θ and R are jointly determined by (10) and (11). These are key equations characterizing the optimal policy in the benchmark model. They ensure that the rate of job creation and job destruction prevailing in the decentralized economy coincide with the planner's optimum.

These conditions have a potentially insightful interpretation. Let us start with the implementation of the optimal level of job creation, (21). Under free entry, firms should only capture a fraction $1-\eta(\theta)$ of the surplus from a match; otherwise, entry is too high and too many resources are allocated to recruitment. However, employers have all the bargaining power and this must be offset by setting a firing tax that exceeds the hiring subsidy in order to reduce job creation to an efficient level. The second term is just a correction in case the planner's discount rate ρ differs from the market interest rate r. If the planner is more patient than market participants, $\rho < r$, then the social value of a new match exceeds the private value perceived by entrepreneurs. This problem is addressed by raising the hiring subsidy for a given firing tax. Condition, (21), could also be seen as a correction for the failure of the Hosios condition to hold. If it did hold, then output maximization would only require $F = H$.

Let us now turn to the interpretation of the equation implementing the optimal level of job destruction, (22). As can be seen from (15), a layoff tax only affects the threshold R if firms discount the future, $r > 0$. Indeed, any match will eventually be destroyed and, hence, by not laying off its worker now, the firm is only postponing the payment of the tax. Thus the relevant cost imposed by the layoff tax is rF, rather than just F.

A firm that dismisses its worker imposes a double externality on the financing of unemployment insurance. First, the worker will qualify for benefits and, second, he will no longer contribute to its funding by paying payroll taxes. The layoff tax should therefore be sufficiently high to ensure that employers internalize these effects. This is the main message of Blanchard Tirole (2008)[17]. The additional insight that is obtained

[17] In fact, in Blanchard Tirole (2008) payroll taxes do not appear as they should optimally be set equal to zero. However, Cahuc and Zylberberg (2008), who propose a generalization to the case where the government needs to raise taxes on income in order to redistribute wealth across heterogeneous individuals, did explicitly have them affecting the level of layoff taxes.

by extending the analysis to a dynamic context is that there is also a social benefit from laying off a worker: it allows a desirable reallocation of this worker from a low to a high productivity job. This is captured by the third term of equation (22) which was given an intuitive interpretation when the optimal allocation was derived, cf. equation (12). This effect reduces the net social cost of dismissal and, hence, the level of the optimal layoff tax. Again, from an output maximization perspective, the condition for optimal job destruction implicitly corrects for the failure of the Hosios condition to hold. If it did hold, then wages would be sufficiently high for this third term to drop out of the equation. Finally, if $\rho = r$, then the option value of keeping the match alive is properly taken into account by firms and therefore does not affect the size of the optimal layoff tax. However, a correction is needed if the planner's discount factor differs from the interest rate. For example, if the planner is more patient than entrepreneurs, $\rho < r$, then the option value is larger for the social planner than for firms and, hence, the layoff tax needs to be raised.

The level of payroll taxes is simply pinned down by the remaining implementability constraint, i.e. by the government budget constraint, (20). Using the fact that, in steady state, the job creation flow is equal to the job destruction flow, $(1-u)\lambda G(R) = u\theta q(\theta)$, we obtain:

$$\tau = \frac{u}{1-u}\left[b - \theta q(\theta)(F-H)\right]. \tag{23}$$

An important insight from this analysis is that the job destruction side of the economy determines the level of layoff taxes, F; while the job creation side determines the difference between layoff taxes and hiring subsidies, $F-H$. Note that this result is fundamentally due to the implementability conditions, (15) and (19), and will therefore remain true in all extensions of the benchmark model. An important implication, which follows from (23), is that the share of unemployment benefits financed from payroll taxes is essentially determined from the job creation side of the economy, a margin that is absent from Blanchard Tirole (2008).

Further insights on the optimal level of payroll taxes could be gained by replacing $F-H$ in (23) by its value from (21), which, after some straightforward rearrangement using (10), yields:

$$\tau = \frac{u}{1-u}\left[b - \theta q(\theta)\left[\frac{1-R}{\rho+\lambda} - \frac{c}{q(\theta)}\right] + \theta q(\theta)\frac{r-\rho}{r+\lambda}\frac{1-R}{\rho+\lambda}\right]. \tag{24}$$

The flow of unemployment benefits, b, constitutes the social cost of having an unemployed worker. The second term represents the corresponding social benefit. Indeed, at rate $\theta q(\theta)$, an unemployed finds a job which generates a social value equal to the expected profits from production net of the recruitment costs. If $r > \rho$, the value of a match to an entrepreneur is smaller than its social value. This should be offset by having sufficiently large hiring subsidies. But this is costly to the government and, hence, payroll taxes need to be raised accordingly.

Since the optimal rate of unemployment should ensure that the social benefits from joblessness is not too distant from its social cost, we expect the first two terms in (24) to be close to each other. In fact, with time discounting, we expect the first term to be slightly larger than the second one since the benefit will only be realized in the future. This intuition is formally confirmed by rewriting the expression for the payroll tax, (24), as:

$$\tau = \frac{\rho}{\rho + \lambda} u \left[\frac{y}{1-u} - R \right] + \frac{r - \rho}{r + \lambda} \lambda G(R) \frac{1 - R}{\rho + \lambda}. \tag{25}$$

This expression is derived in Appendix A. Hence, without time discounting, i.e. $\rho = r = 0$, payroll taxes are not part of the first-best policy. In this case, both unemployment insurance and hiring subsidies should be financed, exclusively, from layoff taxes.

The intuition is that the optimal rate of unemployment is such that the social cost is equal to the social benefit of having an unemployed worker. The key element is that, with free entry and zero bargaining power to workers, the social benefit is entirely captured by the government as fiscal revenue. Similarly, the social cost, i.e. the unemployment benefits, is a government expense. Hence, the two cancel out of the budget constraint and payroll taxes could be set equal to zero.

The optimal policy could now be fully characterized.

Proposition 1 *When workers are wage takers, the first-best allocation could be implemented by choosing the policy instruments b, H, F and τ that satisfy equations (9), (21), (22) and (25).*

Knowing that the first-best allocation is implementable, we could derive the equilibrium rate of unemployment by setting $\dot{u} = 0$ in the equation determining the dynamics of unemployment, (6a). This yields the well known expression:

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}. \tag{26}$$

This equation nevertheless has an interesting new interpretation in this framework. Whereas, for optimal values of θ and R, this is the *output maximizing rate of unemployment*[18] with risk-neutral workers; here, given the microfoundations laid in terms of risk-averse workers, this is the *optimal rate of unemployment.* Not only could unemployment be too low from an output maximization perspective, it could also be too low from a welfare point of view, which is conceptually very different.

[18] This is often referred to as the "efficient rate of unemployment" in the search-matching literature with risk-neutral workers.

4 Financing of Public Expenditures

A characteristic of employment protection in the proposed framework is that it generates some revenue to the government. Thus, a natural question to ask is whether layoff taxes should be higher when governmental expenditures are higher. This question is particularly interesting in a second-best environment where the financing of public expenditures distorts the labor supply decision of workers. I therefore add a participation margin to the previous model.

People who choose to remain out of the labor force enjoy a dollar value of leisure equal to l. The distribution of l across agents in the economy is given by the c.d.f. $K(l)$. Thus, there exists a threshold $\bar{l}$ such that agents choose to work if and only if their value of leisure l is smaller or equal to $\bar{l}$. In a decentralized economy, the value of the threshold $\bar{l}$ is privately chosen by workers.

4.1 Optimal Allocation

As in the previous section, I begin by determining the optimal allocation of resources. The population is normalized to 1. Let I denote the number of people out of the labor force, N the number of employed workers and U that of unemployed. We clearly have $1 = I + N + U$ and $I = 1 - K(\bar{l})$. Thus, $N + U = K(\bar{l})$. The optimal allocation is the solution to:

$$\max_{\{\theta,R,b,w,\bar{l}\}} \int_0^\infty e^{-\rho t} \left[Nv(w) + \left[K(\bar{l}) - N\right] v(z+b) + \int_{\bar{l}}^\infty v(l) dK(l) \right] dt \tag{27}$$

$$\text{subject to} \quad \dot{N} = \theta q(\theta) \left[K(\bar{l}) - N\right] - \lambda G(R) N \tag{28a}$$

$$\dot{Y} = \theta q(\theta) \left[K(\bar{l}) - N\right] + \lambda N \int_R^1 s dG(s) - \lambda Y \tag{28b}$$

$$Nw + \left[K(\bar{l}) - N\right] b = Y - c\theta \left[K(\bar{l}) - N\right] - E \tag{28c}$$

where Y stands for aggregate production and E for the resources allocated to the public expenditures. It is assumed that non-participating workers are not eligible for unemployment benefits.[19] Note that the dynamic evolution of employment N is used as a constraint, (28a), instead of that of unemployment U. In fact, here, the number of unemployed, $U = K(\bar{l}) - N$, is not a state variable as non-working agents who decide to enter the labor force have to transit through unemployment. Conversely, with less than full insurance, marginal workers who decide to leave the labor force must be unemployed.

[19] This assumption, which is standard in the search-matching literature with endogenous participation and unemployment compensation (see, for instance, Sattinger 1995 and Garibaldi Wasmer 2005), is consistent with job search being observable and the associated absence of moral hazard.

The above formulation implicitly assumes that this is still the case with perfect insurance. In other words, U is not a state variable as it jumps when the control variable $\bar{l}$ jumps.

The optimality conditions are identical to those of the previous section. Perfect insurance is still desirable, which combined with the resource constraint (28c), gives:

$$K(\bar{l})w = Y - c\theta\left[K(\bar{l}) - N\right] - Nz - E, \quad (29)$$

$$K(\bar{l})b = Y - c\theta\left[K(\bar{l}) - N\right] + \left[K(\bar{l}) - N\right]z - E. \quad (30)$$

The optimal values of θ and R are still determined by equations (10) and (11). The only novelty is the condition for the optimal participation threshold $\bar{l}$, which in steady state, is:

$$\frac{v(w) - v(\bar{l})}{v'(w)} = \frac{\rho}{\rho + \lambda}\left[(1-R)G(R) + \int_R^1 (s-R)dG(s)\right]\frac{N}{K(\bar{l})} - \frac{E}{K(\bar{l})}. \quad (31)$$

Without public expenditures, $E = 0$, and with perfect insurance, we would expect to obtain $\bar{l} = w = z + b$. But, as can be seen from the first term on the RHS of (31), such is not the case when the planner discounts the future, i.e. when $\rho > 0$. The intuition for $w = z + b > \bar{l}$ is that, initially, when a person enters the labor force, he becomes unemployed and qualifies for unemployment benefits, which is costly to the government, while he will only become productive in a more distant future. Conversely, if we had assumed that the transition was directly from outside the labor force to employment, without intervening unemployment, we would have obtained $w = z + b < \bar{l}$ since, in this case, the marginal worker is producing and therefore relaxes the resource constraint, (28c). Anyway, the first term of the RHS of (31) is not very interesting for our purpose and would vanish by assuming either $\rho = 0$ or that workers enter the labor force with a probability u of being unemployed and $1 - u$ of being employed, where $u = (K(\bar{l}) - N)/K(\bar{l})$ denotes the rate of unemployment.

When $E > 0$, the interesting term in (31) is the last one. When some public expenditures need to be financed, it is desirable to have a larger share of the population working, $\bar{l} > w = z + b$. This increases the number of households who contribute to the financing of the government expenditures. In other words, the social value of participation, $\bar{l}$, is larger than the private value that a worker derives, $w = z + b$. The failure of workers to internalize the entire social value of their participation decision explains why, as we shall see, it is not possible to implement a first-best allocation of resources in a decentralized economy.

4.2 Optimal Policy

I now turn to the determination of the optimal policy in an economy where workers have no bargaining power, i.e. where $w = z + b$. The implementability constraints for job destruction and job creation are the same as before, i.e. (15) and (19), respectively. Public expenditures, E, should be added to the government budget constraint which then becomes:

$$N\tau + N\lambda G(R)F = \left[K(\bar{l}) - N\right] b + \left[K(\bar{l}) - N\right] \theta q(\theta) H + E. \tag{32}$$

The novelty is that workers privately choose whether to participate or not and the government cannot influence this decision by taxing the leisure of non-participating individuals. Thus, workers will only participate if their value of leisure, l, is lower than the income they get while participating. This yields a new implementability constraint for $\bar{l}$ which, under perfect insurance, is[20]:

$$\bar{l} = z + b. \tag{33}$$

But, this cannot be reconciled with the first-best choice of $\bar{l}$ given by equation (31). Hence, the first-best allocation is not implementable here.

The optimal policy is instead derived by adding the implementability constraints to the planner's problem. Now, (27) should be maximized under the previous constraints (28a), (28b), (28c), the equilibrium wage when workers have no bargaining power, i.e. $w = z + b$, and the binding implementability constraint (33). This yields the optimal second-best policy. Strictly speaking, the other implementability constraints, (15), (19) and (32), should also be included. However, they can be safely omitted as they form a system of three equations in three unknowns, τ, F and H, which do not appear elsewhere in the problem.

I have just described how the optimal policy should be derived when workers have no bargaining power. But note that, in a second-best environment, it is not clear that perfect insurance is still desirable. Hence, the corresponding policy might not be second-best but third-best.[21] To check this, the above problem should be solved without imposing any restriction on the net wage w, which could then be treated as a control variable. Importantly, the implementability constraint for $\bar{l}$ needs to be changed; (33) should now be replaced by:

$$v(\bar{l}) = \frac{(\rho + \lambda G(R))\, v(z + b) + \theta q(\theta) v(w)}{\rho + \lambda G(R) + \theta q(\theta)}, \tag{34}$$

which says that the marginal worker's utility from not participating must be equal to the expected utility from unemployment. It turns out that, with no discounting, $\rho = 0$, perfect insurance is still desirable. With discounting, $\rho > 0$, insurance should be less than perfect in order to deter the entry of new workers who would initially all be unemployed and would all qualify for unemployment benefits. This is related to the first

[20] It is implicitly assumed that the leisure value of unemployment, z, is sufficiently low so that the solution to the problem is well-behaved and non-trivial.

[21] This assumes that the government can increase the bargaining power of workers if it is optimal to do so.

term on the RHS of equation (31), which, as previously argued, is not really interesting. What is important is that, as far as the government expenditures E are concerned, the impossibility of implementing the first-best level of participation does not justify any departure from perfect insurance. This is intuitive since the suboptimally low level of participation is due to the existence of a wedge between the social and the private return from work which can only be worsen by under-providing insurance to workers.

Let us now turn to the characteristics of the optimal policy when workers are wage takers. Under perfect insurance, the level of benefits b is still given by equation (30) which in steady state, $\dot{N} = \dot{Y} = 0$, simplifies to:

$$b = y - c\theta u - (1-u)z - \frac{E}{K(\bar{l})}, \tag{35}$$

where u denotes the rate of unemployment and y the level of output per participant, i.e. $Y/K(\bar{l})$. It turns out that the optimal value of the threshold R and market tightness θ are still determined by the first-best conditions (10) and (11). The implementability constraints for job creation and job destruction being the same as before, i.e. (15) and (19), the optimal level of hiring subsidies H and layoff taxes F are still given by (21) and (22). Finally, the level of payroll taxes is determined by (32) which, in steady state, could be written as:

$$\begin{aligned} \tau &= \frac{u}{1-u}\left[b - \theta q(\theta)\left(F-H\right)\right] + \frac{E}{N} \\ &= \frac{u}{1-u}\left[y - c\theta u - (1-u)z - \theta q(\theta)\left(F-H\right)\right] + \frac{E}{K(\bar{l})}, \end{aligned} \tag{36}$$

where the second line was derived by substituting expression (35) for the optimal level of unemployment benefits.

Clearly, from (35) and (36), $b + \tau$ is unaffected by the level of public expenditures. Hence, from (22), layoff taxes remain unchanged; furthermore, from (21), hiring subsidies also remain unchanged. This leads to the following proposition:

Proposition 2 *The amount of public expenditures, E, has no effect on the optimal level of layoff taxes and hiring subsidies.*

The public expenditures are entirely financed through higher payroll taxes and lower unemployment benefits. This result might seem surprising as, in a second-best environment, intuition suggests that two small distortions are preferable to a single large one. This should have led us to expect that the public expenditures should be partly financed from layoff taxes. Such is not the case. In fact, this is a consequence of the Diamond-Mirrlees (1971) production efficiency result according to which optimal taxes never lead to any deviation from production efficiency as this would add some distortions without correcting the existing ones. This result applies since the rate of job creation and job destruction could be seen as being part of the aggregate production function of the economy. Hence, layoff taxes and hiring subsidies should be viewed as Pigouvian instruments used to correct for externalities induced by the decisions of entrepreneurs, not as a general source of revenue for the government.[22]

5 Limits to Insurance

It has so far been assumed that workers could be perfectly insured against the risk of becoming unemployed. Following Blanchard Tirole (2008), I now consider the possibility that there is a non-insurable utility cost $B > 0$ of unemployment. This specification is consistent with findings from the happiness literature which has provided extensive evidence that unemployment has a long-lasting negative effect on life satisfaction; see, for example, Clark Diener Georgellis Lucas (2008). The social planner's problem now becomes:

$$\max_{\{\theta,R,b,w\}} \int_0^\infty e^{-\rho t} \left[(1-u)v(w) + u\left[v(z+b) - B\right]\right] dt \tag{37}$$

$$\text{subject to} \quad \dot{u} = \lambda G(R)(1-u) - \theta q(\theta) u \tag{38a}$$

$$\dot{y} = \theta q(\theta) u + \lambda(1-u)\int_R^1 s dG(s) - \lambda y \tag{38b}$$

$$(1-u)w + ub = y - c\theta u \tag{38c}$$

where the constraints remain unchanged. Equations (8), (9) and (10) still characterize the optimal allocation. Importantly, it remains desirable to equalize the marginal utility of consumption across different states and, hence, to have $w = z + b$. Thus, B is said to be non-insurable as it does not affect marginal utilities and should therefore not be compensated by higher consumption during unemployment. The only difference to the optimal allocation is that the condition for optimal job destruction, (11), is replaced by:

$$R = z + \frac{\eta(\theta)}{1-\eta(\theta)} c\theta - \frac{B}{v'(w)} - \frac{\lambda}{\rho+\lambda}\int_R^1 (s-R) dG(s). \tag{39}$$

Now that workers cannot be perfectly insured against unemployment, it is desirable to decrease the threshold productivity below which a job is destroyed.

Implementing the optimal wage is not as straightforward as before. Indeed, if workers have zero bargaining power, their wage rate is determined by $v(w) = v(z+b) - B$,

[22] The proposition might seem to contradict the findings of Cahuc and Jolivet (2003) who show that public expenditures increase the optimal size of layoff taxes. However, their model does not allow for government-provided unemployment insurance and the increase in layoff taxes is fully compensated by an increase in hiring subsidies. Hence, the public expenditures are entirely financed from taxes on income.

which is not desirable as the marginal utility of consumption would then be higher when employed than when unemployed. The optimal policy could nevertheless be implemented when workers have sufficiently low bargaining power by setting a binding minimum wage equal to $z + b$.[23] Or, alternatively, if the wage rate is exogenously fixed such as to satisfy the resource constraint (38c), by enforcing the optimal level of unemployment benefits given by (9).

Since the implementability constraints for job destruction (15), job creation (19) and the government budget constraint (20) are not affected by the utility cost of being unemployed, it is straightforward to derive the optimal policy. $F - H$ remains given by (21) and τ by (23). The only modification is that F now solves:

$$rF = b + \tau - \frac{\eta(\theta)}{1-\eta(\theta)}c\theta + \frac{B}{v'(w)} + \frac{r-\rho}{r+\lambda}\frac{\lambda}{\rho+\lambda}\int_R^1 (s-R)dG(s). \tag{40}$$

Layoff taxes need to be raised[24] in order to implement the new optimal threshold which is lower than before. Although a similar result has already been derived by Blanchard and Tirole (2008), the interpretation is slightly richer in a dynamic context. The optimal policy implements a lower productivity threshold R and, hence[25], a higher market tightness θ. This induces a decline in the rate of job destruction, $\lambda G(R)$, and a rise in the rate of job creation, $\theta q(\theta)$, which unambiguously leads to a lower equilibrium rate of unemployment. It is interesting to note that the optimal job creation condition (10) is only indirectly affected, through R, by the non-insurable utility cost of being unemployed B. This suggests that the planner primarily tries to reduce job destruction while leaving job creation unchanged. This is implemented by an increase in layoff taxes together with a corresponding adjustment in hiring subsidies such as to restore an optimal rate of job creation.

The key new feature of the optimal policy is summarized in the following proposition.

Proposition 3 *A higher non-insurable utility cost of being unemployed, B, is associated with a lower optimal rate of unemployment.*

When insurance cannot be perfect, reducing the number of jobless is a substitute to the provision of unemployment benefits.[26] This policy nevertheless comes at a cost as

[23] Hungerbuhler and Lehmann (2009) argue, in a redistributive context, that the minimum wage could be a useful policy instrument when workers have insufficient bargaining power.

[24] It could be shown that, under the optimal policy, $F = \frac{1}{r+\lambda}\left[[1-R]G(R) + \int_R^1 (s-R)dG(s)\right]$. Hence, strictly speaking, F is decreasing in R if and only if $g(R)\,[1-R] < 1$. For example, this condition is always satisfied for a uniform distribution of idiosyncratic shocks.

[25] If the elasticity of the matching function is not constant, a sufficient condition for θ to be increasing in B is $d\eta(\theta)/d\theta > -\eta(\theta)\,[1-\eta(\theta)]\,/\theta$. This could be seen by totally differentiating the optimal job creation condition (10) with respect to B and by using the fact that $dR/dB < 0$.

[26] This is reminiscent of the over-employment result of the implicit contract literature; see Baily (1974a) and Azariadis (1975).

the lower threshold R hinders the reallocation of workers from low to high productivity jobs and, hence, net output is no longer maximized. It follows that purchasing power is now lower for both the employed and the unemployed. This case clearly highlights the conceptual distinction between the *output maximizing rate of unemployment* and the welfare maximizing *optimal rate of unemployment.*

Finally, it is possible to compute the optimal level of payroll taxes by replacing b and $F - H$ by their optimal values in the steady state government budget constraint, (23). This yields:

$$\tau = -u\frac{B}{v'(w)} + \frac{\rho}{\rho+\lambda}u\left[\frac{y}{1-u} - R\right] + \frac{r-\rho}{r+\lambda}\lambda G(R)\frac{1-R}{\rho+\lambda}. \tag{41}$$

With no discounting. i.e. $\rho = r = 0$, payroll taxes are negative. The intuition is that the social cost of unemployment now exceeds the corresponding budgetary cost to the government as, without perfect insurance, the social cost of having an unemployed worker is larger than the level of benefits to which he qualifies. However, the social planner still equates the social cost to the social benefit of unemployment and, hence, the budgetary benefit, $\theta q(\theta)(F - H)$, now exceeds the budgetary cost, b. This generates a surplus that allows the implementation of negative payroll taxes or, equivalently, of positive employment subsidies.

6 Workers with Bargaining Power

Under risk aversion, it is desirable to suppress any fluctuations in income between employment and unemployment. Thus, the implementation of a first-best allocation requires workers to have zero bargaining power, as stated in Lemma 1. However, it could be objected that workers fundamentally do have some bargaining power and that this cannot be influenced by the planner. Thus, when solving for the optimal policy, the expression for the wage rate resulting from the bargaining process should be added to the implementability constraints. The resulting planner's problem yields first-order conditions which are hardly interpretable. Hence, I perform a reasonable calibration of the model and report numerical evaluations of the optimal policy for different values of the bargaining power of workers.

An obvious limitation of the analysis of this section is that it does not allow for private savings. When workers have some bargaining power, their income fluctuates over time which should induce them to borrow and save through a risk-free asset in order to smooth their consumption over time. It should nevertheless be acknowledged that, in practice, workers are often liquidity constrained, as shown by Card Chetty Weber (2007) and Chetty (2008), and that assuming unrestricted risk-free borrowing and lending might be even more remote from reality than assuming that workers have to consume their cash-on-hand at each instant.

6.1 No Commitment: Surplus Splitting

With bargaining, wages typically depend on worker's outside opportunities which are affected by a number of endogenous parameters. In order to address these effects, I first propose to implement the optimal policy in a decentralized economy where wages are determined by surplus splitting as in Mortensen-Pissarides (1994, 2003). Thus, workers get a proportion β of the dollar amount of the surplus from the match. It could, fairly, be objected that worker's risk aversion should be explicitly taken into account in the wage bargaining process. However, in the absence of commitment, the resulting bargaining problem would be intractable. Thus, surplus splitting could seen as a proxy for the outcome of the wage bargaining process without commitment. Also, splitting the surplus in fixed proportions does not seem completely implausible[27] and has the important advantage of yielding closed form solutions for the wage rates. This transparently shows how wages are affected by the endogenous variables of the model.

Wages are bargained over each time a productivity shock occurs. The initial net wage, denoted $w_0(1)$, is different from others since, in case no agreement is reached, the firm does not receive the hiring subsidy but does not have to pay the firing tax[28]. By contrast, subsequent bargaining is not affected by the subsidy, which is sunk, but does respond to the cost of laying off a worker. The resulting net wage is denoted by $w(x)$ for a match of productivity x. The corresponding expressions are:

$$w_0(1) = \beta\left[1 + c\theta - \tau - \lambda F + (r+\lambda)H\right] + (1-\beta)\left[z+b\right], \quad (42)$$

$$w(x) = \beta\left[x + c\theta - \tau + rF\right] + (1-\beta)\left[z+b\right], \quad (43)$$

where it is assumed that workers and firm both discount future income at rate r. Details on the surplus splitting rules and on the value functions of workers and firms used to derive these expressions are given in Appendix B.[29] An attractive feature of these wage rates is that they capture the fact that, initially, the hiring subsidy increases the bargaining power of workers while the firing tax decreases it; while, subsequently, the hiring subsidy is sunk and the firing tax put workers in a stronger position. Also, importantly, a higher market tightness reduces the length of unemployment which improves the outside option

[27] This is indeed the form of wage bargaining that was considered by Blanchard and Tirole (2008) in an extension to their benchmark model.

[28] The layoff tax nevertheless enters the expression for the initial wage rate as it affects the firm's expected profits from a newly created match.

[29] Also, note that similar expressions are carefully derived in Mortensen Pissarides (2003) and in Pissarides (2000, chapter 9).

of workers and, hence, their wages.

Proceeding as in the first section, it is easy to show that the job destruction condition, determined by $J(R) = -F$, is now given by:

$$R = z + b + \tau + \frac{\beta}{1-\beta}c\theta - rF - \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s); \tag{44}$$

while the job creation condition, resulting from free entry $V = 0$, is:

$$(1-\beta)\left[\frac{1-R}{r+\lambda} + H - F\right] = \frac{c}{q(\theta)}. \tag{45}$$

Note that these two expressions generalize the previous implementability conditions. Indeed, for $\beta = 0$, (44) and (45) reduce to (15) and (19), respectively.

With fluctuating wages, it is clearly impossible to implement the first-best allocation. The optimal policy should therefore be solved directly under the implementability constraints, i.e. under the decentralized job destruction, (44), and job creation, (45), conditions and under the government budget constraint, (20). The corresponding optimization problem is:

$$\max_{\{\theta,R,b,\tau,F,H\}} \int_0^\infty e^{-\rho t}\left[nv(w_0(1)) + (1-u-n)\int_R^1 \frac{v(w(x))}{1-G(R)}dG(x) + uv(z+b)\right]dt \tag{46}$$

$$\begin{aligned}
\text{subject to}\quad & \dot{u} = \lambda G(R)(1-u) - \theta q(\theta)u && \text{(47a)}\\
& \dot{n} = \theta q(\theta)u - \lambda n && \text{(47b)}\\
& \dot{y} = \theta q(\theta)u + \lambda(1-u)\int_R^1 sdG(s) - \lambda y && \text{(47c)}\\
& nw_0(1) + (1-u-n)\int_R^1 \frac{w(x)}{1-G(R)}dG(x) + ub = y - c\theta u && \text{(47d)}\\
& R = z + b + \tau + \frac{\beta}{1-\beta}c\theta - rF - \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s) && \text{(47e)}\\
& (1-\beta)\left[\frac{1-R}{r+\lambda} + H - F\right] = \frac{c}{q(\theta)} && \text{(47f)}\\
& (1-u)\tau + (1-u)\lambda G(R)F = ub + u\theta q(\theta)H && \text{(47g)}
\end{aligned}$$

where n denotes the number of matches which have not been hit by an idiosyncratic shock yet and with prevailing wage $w_0(1)$. The second constraint, (47b), depicts the dynamics of n. Clearly, the expressions for the wage rate, (42) and (43), should be substituted into the maximization problem where needed. As the resulting first-order conditions are extremely heavy and hardly interpretable, I now rely on a numerical calibration of the model.

I use the same functional forms and parameter values as in Mortensen Pissarides (2003), except for risk aversion which does not appear in their model. Thus, I take a Cobb-Douglas matching function, which reduces to:

$$q(\theta) = q_0 \theta^{-\eta}. \tag{48}$$

It clearly implies that the matching function has a constant elasticity, η. The distribution of idiosyncratic shocks is assumed to be uniform on $[\psi, 1]$; hence its c.d.f. is:

$$G(x) = \frac{x - \psi}{1 - \psi}. \tag{49}$$

Finally, I use a standard constant relative risk aversion (CRRA) instantaneous utility function with CRRA coefficient ϕ:

$$v(x) = \frac{x^{1-\phi}}{1 - \phi}. \tag{50}$$

The chosen exogenous parameter values are displayed in Table 1, where the unit of time is a quarter.[30]

Table 1: Exogenous parameter values

r	ρ	η	c	z	λ	ψ	q_0	ϕ
0.02	0.02	0.5	0.3	0.35	0.1	0.65	1	3

The calibration results are reported for four different values of the bargaining power of workers, β. The initial case, $\beta = 0$, corresponds to the first-best benchmark. The

[30] When $\beta = \eta$ and with no government intervention other than the provision of some unemployment benefits, $b = 0.2$, entirely financed from payroll taxes, the chosen calibration implies that the equilibrium rate of unemployment, u, is 6.56%, the expected length of unemployment, $1/\theta q(\theta)$, is 0.91 quarter and the expected duration of a match, $1/\lambda G(R)$, is 12.93 quarters. These values are within the empirically plausible range reported by Shimer (2007).

results are displayed in Table 2.

Table 2: Optimal policy under surplus splitting

β	0	0.25	0.5	0.75
θ	1.88	1.39	0.66	0.24
R	0.901	0.897	0.878	0.833
u (%)	4.98	5.64	7.42	9.72
n	0.682	0.665	0.602	0.473
y	0.937	0.929	0.906	0.867
Average Wage	0.926	0.934	0.934	0.918
b	0.576	0.434	0.365	0.323
τ	0.0007	-0.0016	-0.0054	-0.0074
F	0.706	0.620	0.594	0.581
H	0.295	0.229	0.0612	-0.225
$F - H$	0.411	0.390	0.533	0.806
Welfare Loss (%)	0	0.36	1.78	4.91
Gross Job Flow	0.0682	0.0665	0.0602	0.0473
$(1-u)\tau/ub$ (%)	2.33	-6.04	-18.55	-21.41

Welfare loss is computed as the proportional decline in consumption in the first-best case necessary to reach the new level of welfare. For example, when $\beta = 0.5$, welfare is equal to what it would be in the first-best allocation, $\beta = 0$, with consumption decreased by 1.78%. In steady state, the gross job flow is given by $u\theta q(\theta)$ or, equivalently, by $(1-u)\lambda G(R)$. Finally, the last row reports the share of unemployment insurance expenses financed by payroll taxes.

When the Hosios condition holds, i.e. when $\beta = 0.5$, the output maximizing policy should not distort job creation or job destruction and, therefore, requires $F = H$. As shown in Table 2, such policy is not welfare maximizing with risk-averse agents. Thus, when workers have some bargaining power, there is a trade-off between output maximization and insurance provision. More precisely, the planner wants to reduce market tightness in order to decrease wages which, by relaxing the resource constraint, allows an increase in the level of unemployment benefits. He therefore set layoff taxes higher than hiring subsidies in order to reduce entry. An additional reason to decrease hiring subsidies is to further reduce the initial wage rate, $w_0(1)$, to which 60% of the workers qualify.

Due to the resource constraint, the level of unemployment benefits decreases with the bargaining power of workers. Also, F is so much higher than H that it generates sufficient surpluses to finance entirely the unemployment benefits as well as some employment subsidies, reported as negative payroll taxes. However, for all values of β, the magnitude of F only corresponds to about two months of the average wage of the economy. This is more than sufficient to pay for the unemployment benefits given that, either, β is low and the expected length of unemployment is short, or, β is high and the replacement ratio is low.

The reservation threshold R declines with bargaining power in order to compensate for the imperfect provision of insurance and for the high length of unemployment induced by the low market tightness. But, this comes at the cost of a more sclerotic labor market characterized by a lower reallocation of workers from low to high productivity jobs, as shown by the lower gross job flow. The reduction in the rate of job creation being larger than that of job destruction, unemployment increases with β. Output, which in steady state can be written as $y = (1-u)\left[G(R) + \int_R^1 s dG(s)\right]$, declines because a smaller number of people work, i.e. unemployment is higher, and the average productivity of employed workers is also reduced due to a lower reservation threshold.

In other words, the downward adjustment in θ and R, which enhances the provision of insurance, hinders the reallocation of workers from low to high productivity jobs, which reduces aggregate output. This is the essence of the trade-off between insurance and production. Also, it should be emphasized that a moderate amount of private savings is likely to reduce, but certainly not to eliminate, the demand for insurance. Thus, a trade-off would remain, albeit of a smaller magnitude, and the key qualitative insights about the optimal policy would presumably remain unaltered.

How would the optimal policy change if wages were re-bargained immediately after recruitment? In this case, newly employed workers would get wage $w(1)$ as given by (43). In order to solve for the optimal policy with immediate wage renegotiation, it is important to note that the implementability condition for job destruction, (44), and the government budget constraint, (20), remain unchanged while the implementability condition for job creation now becomes:

$$(1-\beta)\frac{1-R}{r+\lambda} + H - F = \frac{c}{q(\theta)}. \tag{51}$$

Thus, the planner's problem is still as above, (46), with $w_0(1)$ from (42) replaced by $w(1)$ from (43) and the job creation condition (47f) replaced by (51). The corresponding simulation results are shown in Table 3.

Table 3: Optimal policy under surplus splitting with immediate wage renegotiation

β	0	0.25	0.5	0.75
θ	1.88	1.39	0.65	0.23
R	0.901	0.899	0.889	0.851
u (%)	4.98	5.70	7.80	10.76
n	0.682	0.672	0.629	0.512
y	0.937	0.929	0.906	0.864
Average Wage	0.926	0.935	0.936	0.924
b	0.576	0.427	0.352	0.302
τ	0.0007	0.0061	0.0146	0.0267
F	0.706	0.539	0.390	0.244
H	0.295	0.263	0.168	0.077
$F-H$	0.411	0.276	0.223	0.168
Welfare Loss (%)	0	0.39	1.93	5.47
Gross Job Flow	0.0682	0.0672	0.0629	0.0512
$(1-u)\tau/ub$ (%)	2.33	23.79	49.06	73.54

The allocation of resources is pretty similar to that of the previous case. The main difference lies in the level of the policy instruments F, H and τ. There are two reasons for that. First, from the implementability condition (51), the difference between hiring subsidies and layoff taxes has a larger impact on job creation than before. Indeed, with immediate renegotiation, these policy instruments have a smaller effect on wages and, hence, a larger effect on firms. This explains why $F-H$ does not need to be as large as before to reduce θ to its desired level. The second reason is that hiring subsidies cease to increase initial wages and layoff taxes cease decrease them. Hence, when workers have a strong bargaining power, it is no longer necessary to maintain high layoff taxes and low hiring subsidies to prevent wages from being too high and unemployment benefits too low. Note that $F-H$ being smaller than before, a significant share of the unemployment benefits now needs to be financed from payroll taxes.

To gain additional insights about the key trade-offs underpinning the optimal policy, let us consider the following naive surplus splitting rule:

$$w(x) = \beta[x-\tau] + (1-\beta)[z+b]. \tag{52}$$

Before going further, it should be emphasized that the intermediary case where $w(x) = \beta[x-\tau+c\theta]+(1-\beta)[z+b]$ is quantitatively almost identical to the immediate renegotiation case as the term rF, in (43), is small. Also note that, for a given allocation, the wage rate is lower under naive surplus splitting, (52), than under immediate renegotiation, (43), as market tightness and layoff taxes cease to have a positive impact. This generates a mechanical improvement in the level of insurance.

When solving for the optimal policy under naive surplus splitting, the implementability conditions remain given by (51) for job creation and by (20) for the government budget constraint while, for job destruction, it becomes:

$$R = z + b + \tau - \frac{rF}{1-\beta} - \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s). \tag{53}$$

The simulation results are presented in Table 4.

Table 4: Optimal policy under naive surplus splitting

β	0	0.25	0.5	0.75
θ	1.88	1.88	1.88	1.88
R	0.901	0.902	0.905	0.909
u (%)	4.98	5.00	5.04	5.11
n	0.682	0.685	0.691	0.701
y	0.937	0.937	0.937	0.938
Average Wage	0.926	0.927	0.928	0.929
b	0.576	0.562	0.549	0.537
τ	0.0007	0.0152	0.0302	0.0453
F	0.706	0.526	0.343	0.169
H	0.295	0.326	0.357	0.387
$F-H$	0.411	0.200	-0.015	-0.221
Welfare Loss (%)	0	0.01	0.02	0.05
Gross Job Flow	0.0682	0.0685	0.0691	0.0701
$(1-u)\tau/ub$ (%)	2.33	51.30	103.66	156.63

Strikingly, market tightness θ and the productivity threshold R are almost independent of the bargaining power of workers. This suggests that, without the general equilibrium effect of market tightness on wages, there is hardly any trade-off between output maximization and insurance provision. Consequently, the main role of layoff taxes and hiring subsidies is to compensate for the failure of the Hosios condition to hold, i.e. to offset the distortions generated by the gap between the bargaining power of workers and the elasticity of the matching function. This explains why, when the Hosios condition does hold, i.e. when $\beta = 0.5$, layoff taxes and hiring subsidies are virtually equal to each other. The slight discrepancy that remains, and which result in payroll taxes covering

103.66% of the cost of providing unemployment insurance, rather than 100%, is due to the positive impact of payroll taxes on wages. Hence, the government tries to increase those taxes a little in order to decrease wages which, through a relaxation of the resource constraint, allows an improvement in the level of unemployment benefits.

6.2 Commitment: Fixed Wage

The previous subsection assumes that the dollar amount of the surplus from the match is split in fixed proportions between the worker and the firm. However, this leads to substantial wage fluctuations which, if firms can commit, seems inconsistent with the risk sharing that would be expected to occur between a risk-averse worker and a risk-neutral employer. In particular, if a firm and a worker discount the future at the same rate, i.e. $r = \rho$, then the firm will commit to paying a fixed wage, w, throughout the duration of the match and to a job destruction threshold, R.

The Bellman equations corresponding to the expected utility of an unemployed, U, and of an employed worker, W, are:

$$rU = v(z+b) + \theta q(\theta)\left[W - U\right], \tag{54}$$

$$rW = v(w) + \lambda G(R)\left[U - W\right], \tag{55}$$

where, as before, $v(.)$ stands for the instantaneous utility of consumption. The two parameters of the contract are determined *ex-ante* by Nash bargaining:

$$\{w, R\} = \arg\max_{\{w_i, R_i\}} \left[W_i - U\right]^{\beta}\left[J_i(1) + H - V\right]^{1-\beta}, \tag{56}$$

where the subscript i is used to stress that the wage and threshold bargained in match i do not affect the value of outside options, i.e. the values of U or V. *Ex-ante* bargaining implies that, if an agreement is not reached, the employer does not receive the hiring subsidy but does not have to pay the layoff tax.

The worker's net salary is determined by:

$$\frac{v(w) - v(z+b)}{v'(w)} = \left[r + \lambda G(R) + \theta q(\theta)\right]\frac{\beta}{1-\beta}\frac{c}{q(\theta)}; \tag{57}$$

while the job destruction threshold solves:

$$R = w + \tau - rF - \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s) - \frac{r + \lambda G(R)}{r + \lambda G(R) + \theta q(\theta)}\frac{v(w) - v(z+b)}{v'(w)}. \tag{58}$$

These two expressions are derived in Appendix C. The last term of the decentralized job destruction condition (58) would not appear without commitment, cf. (15). This shows that firms use both margins to provide insurance to risk-averse workers: they pay a constant wage and they lower the job destruction threshold. Using the free-entry condition, it could easily be shown that the decentralized job creation condition is:

$$(1-\beta)\left[\frac{1-R}{r+\lambda}+H-F\right]=\frac{c}{q(\theta)}. \tag{59}$$

The optimal policy could then be derived by adding the wage equation (57) as a constraint to the original problem. Thus, the planner should maximize (5) with respect to θ, R, b and w subject to (6a), (6b), (6c) and (57). The three remaining implementability constraints, (58), (59) and (20), could be left out since they jointly determine F, H and τ which do not appear elsewhere in the planner's problem. Table 5 displays the simulation results for the same calibrating of the model as before.

Table 5: Optimal policy under Nash bargaining with risk aversion

β	0	0.25	0.5	0.75
θ	1.88	1.59	1.06	0.52
R	0.901	0.898	0.886	0.854
u (%)	4.98	5.32	6.14	7.52
y	0.937	0.933	0.921	0.897
w	0.926	0.933	0.937	0.933
b	0.576	0.448	0.361	0.294
τ	0.0007	0.0007	0.0014	0.0034
F	0.706	0.600	0.485	0.315
H	0.295	0.255	0.155	−0.037
$F-H$	0.411	0.345	0.330	0.352
Welfare Loss (%)	0	0.24	1.07	3.19
Gross Job Flow	0.0682	0.0671	0.0633	0.0540
$(1-u)\tau/ub$ (%)	2.33	2.92	5.76	14.04

Again, the case $\beta=0$ corresponds to the implementation of the first-best policy.

As β increases, θ and R both decline in order to partially offset the increase in the gap between w and $b+z$. Indeed, a higher market tightness puts workers in a stronger bargaining position which is detrimental to insurance. Also, a lower reservation threshold improves the welfare of employed workers and can be compensated by a smaller wage rate. The decline in the rate of job creation being stronger than that of job destruction, unemployment increases with β. Output falls. Due to the resource constraint, the level of unemployment benefits decreases with β.

When β is low, F is higher than H in order to compensate for the failure of the

Hosios condition to hold. As β increases, this becomes a smaller concern, but insufficient insurance becomes a bigger one. The planner therefore wants to decrease market tightness which becomes the main reason why F exceeds H.

Also, layoff taxes are rapidly declining in β and are lower than in the surplus splitting counterpart to this problem, cf. Table 2. The reason is that, as could be seen from (58), firms spontaneously decrease the destruction threshold R whenever insurance is less than perfect. Thus, layoff taxes have a smaller job to do to reduce the rate of job destruction to its optimal level. The surpluses generated by $F - H$ nevertheless remain sufficiently large to finance almost all the unemployment benefits but leave no room for employment subsidies.

The wage and threshold could be determined by directed search, rather than by Nash bargaining. In such an environment, competitive market makers jointly choose the wage rate, the threshold and the length of queues, equal to $1/\theta q(\theta)$, such as to maximize the expected utility of an unemployed worker subject to a free entry condition for firms; or more formally:

$$\max_{\{\theta,w,R\}} \rho U \text{ subject to } V = 0. \tag{60}$$

This yields exactly the same equations as (57) and (58) with β replaced by η. Thus, in Table 5, directed search corresponds to the case where $\beta = \eta = 0.5$. As implied by Corollary 1, directed search and the associated Hosios condition fail to implement a first-best allocation of resources in an economy with risk-averse workers as they fail to ensure a sufficient provision of insurance.

7 Moral Hazard

When workers have some bargaining power, there is typically a trade-off between output maximization and insurance provision. But, reducing the level of insurance might be a virtue if it increases the search intensity of unemployed workers. Indeed, concerns about the moral hazard effects of unemployment insurance have been at the heart of the literature on the topic. Hence, this section characterizes the optimal policy when job search monitoring is not available and, hence, when the unemployed freely choose their search intensity.

7.1 Determination of Search Intensity

Let s denote the search intensity of the unemployed. Vacant jobs and unemployed workers now get matched at rate[31]:

$$m = m(su, v), \tag{61}$$

where the matching function satisfies the same properties as before. Vacancies become filled at rate:

$$\frac{m(su, v)}{v} = m\left(\frac{s}{\theta}, 1\right) = q(\theta, s), \tag{62}$$

where market tightness remains defined as the ratio of vacancies to unemployment, i.e. $\theta = v/u$.[32]

Unemployed worker i who searches with intensity s_i finds a job at rate:

$$\begin{aligned} \tilde{q}(\theta, s, s_i) &= \frac{s_i}{s}\frac{m(su, v)}{u} \\ &= \frac{s_i}{s}\theta q(\theta, s). \end{aligned} \tag{63}$$

The Bellman equation associated with the expected utility of an unemployed worker is:

$$\rho U = v(z + b) - \sigma(s_i) + \tilde{q}(\theta, s, s_i)\left[W(1) - U\right], \tag{64}$$

where σ denotes an increasing and convex cost of search, with $\sigma(0) = \sigma'(0) = 0$, and $W(1)$ is the value of a new job to a worker. The first-order condition for search intensity is:

$$-\sigma'(s_i) + \frac{\partial \tilde{q}(\theta, s, s_i)}{\partial s_i}\left[W(1) - U\right] = 0. \tag{65}$$

Hence, using the symmetry which prevails in equilibrium, i.e. $s_i = s$, the search intensity of unemployed workers is implicitly determined by:

$$s\sigma'(s) = \theta q(\theta, s)\left[W(1) - U\right]. \tag{66}$$

7.2 Surplus Splitting

The optimal policy with moral hazard could now be solved numerically. For this, I focus on the case where wages are determined by surplus splitting as this is the most transparent situation about the influence of the different parameters on wages.

As before, I consider the wage rate that would prevail under surplus splitting if workers

[31] The intensity of job advertising made by firms with a vacancy is exogenously set to 1 as, even if endogenously determined, it would not be affected by any policy parameters; cf. Pissarides (2000, chapter 5.3).

[32] Note that, by definition of the elasticity of the matching function η, $\frac{\partial q(\theta,s)}{\partial \theta} = -\frac{q(\theta,s)}{\theta}\eta(\theta, s)$ and, hence, from (62), $\frac{\partial q(\theta,s)}{\partial s} = \frac{q(\theta,s)}{s}\eta(\theta, s)$.

and firms were both risk-neutral. This gives:

$$w_0(1) = \beta\left[1 + c\theta - \tau - \lambda F + (r+\lambda)H\right] + (1-\beta)\left[z + b - \sigma(s)\right], \tag{67}$$

$$w(x) = \beta\left[x + c\theta - \tau + rF\right] + (1-\beta)\left[z + b - \sigma(s)\right], \tag{68}$$

where the initial wage, $w_0(1)$, applies until a shock occurs. The existence of the search cost $\sigma(s)$ lowers the value of unemployment, which is the outside option, and hence adversely affects wages.

Under these wage rates, the search intensity of a risk-averse worker is determined by:

$$s\sigma'(s) = \theta q(\theta, s)\frac{E\left[v(w)\right] - v(z+b) + \sigma(s)}{\rho + \lambda G(R) + \theta q(\theta, s)}, \tag{69}$$

where the average utility of employed workers is given by:

$$E\left[v(w)\right] = \left[1 - \frac{\lambda}{\rho+\lambda}\left[1 - G(R)\right]\right] v(w_0(1)) + \frac{\lambda}{\rho+\lambda}\int_R^1 v(w(x))dG(x). \tag{70}$$

These expressions are derived in Appendix D. The planner's problem is as before, (46), with s as a new control variable and (69) as an additional constraint.[33]

For reference, I also solve for the optimal policy when the planner is able to freely set the wage of workers. Absent any constraints on the expression for the wage rate, this gives the best possible allocation that could be attained with endogenous search intensity. In that context, the first-order condition for search intensity is (69) with $E[v(w)]$ simply replaced by $v(w)$ where w is the wage chosen by the planner. Also, with a fixed wage, the decentralized job destruction and job creation conditions are given by (15) and (19), respectively.

Before solving for the optimal policy, it is necessary to recalibrate the version of the model which allows for moral hazard. The calibration is done in a context where $\beta = \eta$ and where the government does not intervene except to provide some unemployment benefits, $b = 0.2$, financed from payroll taxes; which is arguably a good sketch of the current U.S. situation. The scale parameter of the matching function, q_0, and the lower bound of the distribution of idiosyncratic shocks, ψ, are set such that the quarterly rates of job creation and job destruction remain equal to 0.91 and 12.93, respectively. This

[33] The other changes are that search intensity should be included in the matching function, i.e. $q(\theta)$ should be replaced by $q(\theta, s)$, and the search cost $\sigma(s)$ should be subtracted for a mass u of unemployed workers from the objective function, i.e. the last term of the objective should be $u\left[v(z+b) - \sigma(s)\right]$ instead of $uv(z+b)$. Finally, z should be replaced by $z - \sigma(s)$ in the decentralized job destruction condition, (47e).

gives $q_0 = 0.83$ and $\psi = 0.49$. The cost of search is assumed to be convex:

$$\sigma(s) = k\frac{s^{\gamma+1}}{\gamma+1}.$$

The constant k is calibrated such that s is normalized to 1 and γ such that the elasticity of unemployment duration with respect to the benefit level is equal to 0.5, a reasonable estimate according to Krueger and Meyer (2002)'s survey of the literature on the topic. This yields $k = 1.16$ and $\gamma = 5.02$. All the other parameters of the model are left unchanged.

The simulation results are presented in Table 7.

Table 7: Optimal policy under surplus splitting and moral hazard

β	Best Wage	0.125	0.2171	0.25	0.5	0.75
θ	2.01	2.32	2.02	1.89	0.94	0.91
R	0.861	0.859	0.862	0.862	0.846	0.835
u (%)	6.50	6.33	6.50	6.64	8.47	4.78
n	0.678	0.675	0.679	0.678	0.635	0.504
y	0.917	0.918	0.917	0.916	0.894	0.915
Average Wage	0.910	0.902	0.910	0.912	0.919	0.933
b	0.418	0.468	0.420	0.406	0.336	0.288
s	0.781	0.707	0.779	0.797	0.867	1.22
τ	−0.0128	0.0008	−0.0006	−0.0013	−0.0062	−0.0036
F	0.997	0.923	0.843	0.822	0.732	0.398
H	0.420	0.495	0.433	0.407	0.194	0.056
$F-H$	0.577	0.428	0.411	0.415	0.538	0.342
Welfare Loss (%)	0	0.198	0.002	0.022	1.279	2.351
Gross Job Flow	0.0678	0.0675	0.0679	0.0678	0.0635	0.0504
$(1-u)\tau/ub$ (%)	−43.82	2.44	−2.17	−4.35	−19.92	−24.78

The first column reports the calibration for the optimal fixed wage, i.e. the "best wage", chosen by the planner. The welfare loss is now computed relative to this benchmark. Thus, for instance, the welfare generated by the optimal policy with surplus splitting when $\beta = 0.5$ is identical to the welfare of the optimal allocation with a fixed wage but with consumption of the employed and unemployed decreased by 1.279%.

When the worker has a low bargaining power, $\beta = 0.125$, market tightness is higher than with the best wage. In fact, the planner wants to increase wages, and reduce insurance, in order to boost the returns to search. Hiring subsidies, which have a positive impact on initial wages, are also set at a very high level. This is exactly the opposite to what would be recommended without moral hazard where market tightness would be reduced in order to improve the provision of insurance.

Welfare is maximized for $\beta = 0.2171$, where the optimal allocation is very similar to that implied by the best wage. Market tightness is nevertheless a little higher which increases the recruitment costs but reduces the provision of insurance which is slightly too high compared to the best wage benchmark. The optimal setting of the policy instruments τ, F and H differs substantially from that of the benchmark. This is due to the differences in the implementability constraints, which are themselves caused by the different specifications of the wage rate.

When $\beta = 0.2171$, the low magnitude of the welfare loss, which is below 0.002%, suggests that, at the optimum, the surplus splitting rule hardly worsens the trade-off between insurance and production, compared to the optimal fixed wage case. Indeed, the forces pushing for more insurance, i.e. risk aversion, and less insurance, i.e. moral hazard, nearly offset each other. Hence, given the prevailing level of insurance, the policy parameters could be set such as to maximize the reallocation of workers from low to high productivity jobs. Indeed, at the optimal β, the reservation threshold R is close to being maximized.

For a higher bargaining power, market tightness does not need to be pushed upward as wages are already sufficiently high to reward search efforts. The previous intuitions, without moral hazard, dominate again and market tightness should be decreased in order to improve the provision of insurance. Thus, for high values of β, the introduction of moral hazard does not really modify the qualitative conclusions reached in the previous section about the key characteristics of an optimal policy.

With immediate wage renegotiation, newly employed worker are paid $w(1)$ as given by (68). The planner's problem is obtained by adding the constraint for search intensity, given by (69) with $w(1)$ replacing $w_0(1)$ in (70), to the corresponding problem of the previous section.[34] The simulated optimal policy is shown in Table 8.

Table 8: Optimal policy with immediate renegotiation and moral hazard

β	Best Wage	0.125	0.2127	0.25	0.5	0.75
θ	2.01	2.27	2.01	1.86	0.93	0.35
R	0.861	0.858	0.863	0.864	0.862	0.840
u (%)	6.50	6.36	6.53	6.70	8.82	12.76
n	0.678	0.673	0.681	0.681	0.663	0.596
y	0.917	0.918	0.917	0.916	0.895	0.850
Average Wage	0.910	0.902	0.910	0.913	0.922	0.918
b	0.418	0.466	0.419	0.402	0.328	0.280
s	0.781	0.710	0.781	0.802	0.876	0.909
τ	−0.0128	0.0035	0.0058	0.0068	0.0170	0.0334
F	0.997	0.891	0.778	0.736	0.496	0.281
H	0.420	0.498	0.455	0.434	0.294	0.170
$F - H$	0.577	0.393	0.323	0.302	0.202	0.111
Welfare Loss (%)	0	0.185	0.003	0.030	1.383	5.311
Gross Job Flow	0.0678	0.0673	0.0681	0.0681	0.0663	0.0596
$(1-u)\tau/ub$ (%)	−43.82	11.00	19.68	23.60	53.72	81.46

The optimal allocation is similar to that without immediate renegotiation, but the optimal setting of the policy instruments is now different. These differences are similar to those between the corresponding tables without moral hazard; see Table 2 and 3. Welfare is maximized for $\beta = 0.2127$. Again, when workers have substantial bargaining power, the introduction of moral hazard does not modify the main conclusions of the previous section as the primary concern of the planner remains the under-provision of insurance to workers.

Finally, to get some further insights, I consider the naive surplus splitting rule:

$$w(x) = \beta[x - \tau] + (1 - \beta)[z + b - \sigma(s)]. \tag{71}$$

[34] Appropriate adjustments for search intensity should be made as described in the previous footnote.

The corresponding optimal policy with moral hazard is reported in Table 9.

Table 9: Optimal policy with naive surplus splitting and moral hazard

β	Best Wage	0.125	0.25	0.5	0.75
θ	2.01	1.73	1.76	1.79	1.81
R	0.861	0.827	0.835	0.848	0.860
u (%)	6.50	7.91	7.65	7.46	7.45
n	0.678	0.605	0.623	0.647	0.669
y	0.917	0.894	0.899	0.904	0.908
Average Wage	0.910	0.881	0.887	0.893	0.897
b	0.418	0.522	0.518	0.508	0.497
s	0.781	0.488	0.544	0.606	0.643
τ	−0.0128	0.0064	0.0172	0.0399	0.0627
F	0.997	1.047	0.861	0.539	0.251
H	0.420	0.463	0.480	0.523	0.564
$F - H$	0.577	0.584	0.381	0.015	−0.313
Welfare Loss (%)	0	1.824	1.291	0.820	0.601
Gross Job Flow	0.0678	0.0605	0.0623	0.0647	0.0669
$(1-u)\tau/ub$ (%)	−43.82	14.33	40.13	97.38	156.56

The key problem of the planner is that naive surplus splitting generates too much insurance and there is hardly any way to undo this as the wage rate is largely independent of the parameters under the planner's control. There is a complementarity between market tightness and search intensity as they both increase the matching rate $\theta q(\theta, s)$. However, given the over-provision of insurance, search intensity is low and it is therefore not worth pushing market tightness upward. Also, since the unemployed are very inefficient at searching for jobs, the reallocation of workers from low to high productivity jobs is long and costly and, hence, the threshold productivity R is reduced as it is now preferable to keep workers in low productivity occupations. However, as β increases, the problem of over-insurance becomes less severe and welfare improves.

As the level of insurance cannot really be influenced, the main effect of layoff taxes and hiring subsidies is to correct for the failure of the Hosios condition to hold. Hence, when $\beta = 0.5$, both are approximately equal to each other.

8 Conclusion

In this paper, I have investigated optimal policies in a dynamic search model with risk-averse workers. More precisely, I have focused on the joint derivation of the optimal level of unemployment benefits, layoff taxes, hiring subsidies and payroll taxes.

I began by abstracting from moral hazard in order to focus on the general equilibrium effects of the different policy instruments. I showed that the first-best allocation of resources can be implemented in a decentralized economy when workers are wage takers. In this situation, full insurance is provided and output is maximized. Layoff taxes are higher than hiring subsidies in order to offset the excessive entry of vacancies caused by the absence of bargaining power of workers. Moreover, the corresponding surplus is sufficiently large to finance nearly all the unemployment benefits and payroll taxes are therefore hardly needed.

However, layoff taxes and hiring subsidies should only be viewed as Pigouvian instruments used to correct externalities, not as a general source of revenue to the government. Indeed, additional public expenditures should be entirely financed through higher payroll, or income, taxes and lower unemployment benefits, even in a second-best environment with endogenous participation.

The analysis being properly microfounded in terms of risk-averse workers, it allows the determination of an optimal, welfare maximizing, rate of unemployment, which goes beyond the well-known output maximizing rate of unemployment. The distinction between the two becomes particularly relevant when there is a trade-off between the provision of insurance and the maximization of production. For instance, the optimal rate of unemployment is lower when workers are confronted with a non-insurable utility cost of unemployment. Intuitively, a reduction in the probability of unemployment is a substitute to the provision of unemployment benefits.

When workers have some bargaining power, the planner wants to reduce wages in order to relax the resource constraint and improve the level of unemployment benefits. In particular, this is achieved by reducing market tightness which lowers wages, as desired, but also hinders the reallocation of workers from low to high productivity jobs. Introducing moral hazard adds a counteracting force to the model. When workers have a very low bargaining power, it is typically desirable to increase market tightness and to boost wages in order to enhance the reward to the search effort of the unemployed. However, when workers have a more substantial bargaining power, under-provision of insurance, rather than moral hazard, remains the primary concern of the planner. Chetty (2008) has already argued that the issue of moral hazard might have been over-emphasized in the literature. The present paper adds to this by showing that general equilibrium effects on job creation, job destruction and wages might be at least as important for the determination of optimal policies.

There are essentially two reasons which could justify setting layoff taxes higher than hiring subsidies; in which case the difference between the two could cover at least some of the costs of providing unemployment benefits. First, to compensate for the failure of the Hosios condition to hold; or, in other words, to reduce entry in order to save on the recruitment costs when the bargaining power of workers is lower than the elasticity of the matching function. Second, in order to reduce wages, by reducing market tightness and hiring subsidies, when the provision of insurance is insufficient. Importantly, as the bargaining power of workers increases, the first reason becomes less relevant while the second becomes more important. This is why layoff taxes exceed hiring subsidies in all realistic calibrations of the model and for any bargaining power of workers.

This shows that, without governmental intervention, labor markets with search frictions generically implement an inefficient allocation of resources. With risk-neutral workers, inefficiencies are only due to unbalanced search externalities associated with deviations from the Hosios condition. Here, the inefficiency is much deeper and involves a lack of insurance against the risk of becoming unemployed.

Some important issues are left for further research. First, an accurate empirical knowledge of the main determinants of wages, at the macroeconomic level, is key for the optimal design of labor market policies.[35] Knowing, quantitatively, how wages are affected by market tightness or by the different policy instruments is obviously essential if the planner wants to increase the provision of insurance at the smallest cost in terms of output. The precise specification of wages also crucially affects the implementability constraints. For instance, if layoff taxes and hiring subsidies are passed on to workers through adjustment in wages, then they have a much smaller effect on the job creation and job destruction decisions of firms.

Throughout this paper, I have only considered time invariant policy instruments. In fact, in a dynamic context, it would be interesting to allow the level of unemployment benefits to be affected by the length of unemployment and that of layoff taxes and hiring subsidies to depend on the age of the match, among other things. Also, in the proposed model, the length of unemployment does not directly matter, only its rate does.[36] This could be relaxed by assuming that the level of human capital depreciates during an unemployment spell[37] or, more simply, by assuming that workers have a preference for shorter spells even if this is associated with a higher probability of being unemployed. The length of unemployment being decreasing in market tightness, the resulting optimal policy would presumably advocate for a smaller reduction in the rate of job creation.

[35] Blanchflower and Oswald (1994) provide extensive evidence of the negative impact of unemployment on wages. However, their work does not control for the number of vacancies and, hence, cannot identify the impact of market tightness on wages.

[36] The length of unemployment nevertheless has an impact on the speed of the reallocation of workers from low to high productivity jobs.

[37] See the related analyses of Pavoni (2008) and Shimer Werning (2006) who determine the optimal unemployment insurance policy with human capital depreciation.

References

[1] Acemoglu, D. and Shimer, R. (1999), 'Efficient Unemployment Insurance', *Journal of Political Economy*, 107(5), 893-928.

[2] Acemoglu, D. and Shimer, R. (2000), 'Productivity Gains from Unemployment Insurance', *European Economic Review*, 44, 1195-1224.

[3] Alvarez, F. and Veracierto, M. (2000), 'Labor-Market Policies in an Equilibrium Search Model', in *NBER Macroeconomics Annual 2000*, edited by Ben S. Bernanke and Kenneth Rogoff, Cambridge, MA: MIT Press.

[4] Alvarez, F. and Veracierto, M. (2001), 'Severance Payments in an Economy with Frictions', *Journal of Monetary Economics*, 47, 477-498.

[5] Azariadis, C. (1975), 'Implicit Contracts and Underemployment Equilibria', *Journal of Political Economy*, 83(6), 1183-1202.

[6] Baily, M.N. (1974a), 'Wages and Unemployment under Uncertain Demand', *Review of Economic Studies*, 41(1), 37-50.

[7] Baily, M.N. (1974b), 'Some Aspects of Optimal Unemployment Insurance', *Journal of Public Economics*, 10, 379-402.

[8] Bentolila, S. and Bertola, G. (1990), 'Firing Costs and Labour Demand: How Bad is Eurosclerosis?', *Review of Economic Studies*, 57(3), 381-402.

[9] Blanchard, O.J. and Tirole, J. (2008), 'The Joint Design of Unemployment Insurance and Employment Protection: A First Pass', *Journal of the European Economic Association*, 6(1), 45-77.

[10] Blanchflower, D.G. and Oswald, A.J. (1994), *The Wage Curve*, Cambridge, MA: MIT Press.

[11] Cahuc, P. and Jolivet, G. (2003), 'Do We Need More Stringent Employment Protection Legislations?', Working Paper, CREST.

[12] Cahuc, P. and Laroque, G. (2009), 'Optimal Taxation and Monopsonistic Labor Market: Does Monopsony Justify the Minimum Wage?', Working Paper, CREST.

[13] Cahuc, P. and Lehmann, E. (2000), 'Should Unemployment Benefits Decrease with the Unemployment Spell?', *Journal of Public Economics*, 77, 135-153.

[14] Cahuc, P. and Malherbet, F. (2004), 'Unemployment Compensation Finance and Labor Market Rigidity', *Journal of Public Economics*, 88, 481-501.

[15] Cahuc, P. and Zylberberg, A. (2008), 'Optimum Income Taxation and Layoff Taxes', *Journal of Public Economics*, 92, 2003-2019.

[16] Card, D., Chetty, R. and Weber, A. (2007), 'Cash-On-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market', *Quarterly Journal of Economics*, 122(4), 1511-1560.

[17] Chetty, R. (2006), 'A General Formula for the Optimal Level of Social Insurance', *Journal of Public Economics*, 90, 1879-1901.

[18] Chetty, R. (2008), 'Moral Hazard versus Liquidity and Optimal Unemployment Insurance', *Journal of Political Economy*, 116(2), 173-234.

[19] Chetty, R. and Saez, E. (2008), 'Optimal Taxation and Social Insurance with Endogenous Private Insurance', Working Paper, UC Berkeley.

[20] Clark, A.E., Diener, E., Georgellis, Y. and Lucas, R.E. (2008), 'Lags and Leads in Life Satisfaction: A Test of the Baseline Hypothesis', *Economic Journal*, 118, F222-F243.

[21] Coles, M. and Masters, M. (2006), 'Optimal Unemployment Insurance in a Matching Equilibrium', *Journal of Labor Economics*, 24(1), 109-138.

[22] Diamond, P.A. and Mirrlees, J.A. (1971), 'Optimal Taxation and Public Production I: Production Efficiency', *American Economic Review*, 61(1), 8-27.

[23] Feldstein, M. (1976), 'Temporary Layoffs in the Theory of Unemployment', *Journal of Political Economy*, 84(5), 937-958.

[24] Fella, G. (2007), 'Optimal Severance Pay in a Matching Equilibrium', Working Paper, Queen Mary.

[25] Fredriksson, P. and Holmlund, B. (2001), 'Optimal Unemployment Insurance in Search Equilibrium', *Journal of Labor Economics*, 19(2), 370-399.

[26] Garibaldi, P. and Wasmer, E. (2005), 'Equilibrium Search Unemployment, Endogenous Participation, and Labor Market Flows', *Journal of the European Economic Association*, 3(4), 851-882.

[27] Hopenhayn, H.A. and Nicolini, J.P. (1997), 'Optimal Unemployment Insurance', *Journal of Political Economy*, 105(2), 412-438.

[28] Hopenhayn, H. and Rogerson, R. (1993), 'Job Turnover and Policy Evaluation: A General Equilibrium Analysis', *Journal of Political Economy*, 101(5), 915-938.

[29] Hosios, A.J. (1990), 'On the Efficiency of Matching and Related Models of Search and Unemployment', *Review of Economic Studies*, 57(2), 279-298.

[30] Hungerbuhler, M. and Lehmann, E. (2009), 'On the Optimality of a Minimum Wage: New Insights from Optimal Tax Theory', *Journal of Public Economics*, 93, 464-481.

[31] Krueger, A.B. and Meyer, B.D. (2002), 'Labor Supply Effects of Social Insurance', in *Handbook in Public Economics*, Volume 4, edited by A.J. Auerbach and M. Feldstein, Amsterdam: North-Holland.

[32] Lehmann, E. and van der Linden, B. (2007), 'On the Optimality of Search Matching Equilibrium When Workers are Risk Averse', *Journal of Public Economic Theory*, 9(5), 867-884.

[33] L'Haridon, O. and Malherbet, F. (2009), 'Employment Protection Reform in Search Economies', *European Economic Review*, 53, 255-273.

[34] Ljungqvist, L. (2002), 'How do Layoff Costs Affect Employment?', *Economic Journal*, 112, 829-853.

[35] Ljungqvist, L. and Sargent, T.J. (2008), 'Two Questions about European Unemployment', *Econometrica*, 76(1), 1-29.

[36] Mongrain, S. and Roberts, J. (2005), 'Unemployment Insurance and Experience Rating: Insurance Versus Efficiency', *International Economic Review*, 46(4), 1303-1319.

[37] Mortensen, D.T. and Pissarides, C.A. (1994), 'Job Creation and Job Destruction in the Theory of Unemployment', *Review of Economic Studies*, 61, 397-415.

[38] Mortensen, D.T. and Pissarides, C.A. (1999), 'New Developments in Models of Search in the Labor Market', in *Handbook in Labor Economics*, Volume 3, Part 2, edited by O. Ashenfelter and D. Card, Amsterdam: North-Holland.

[39] Mortensen, D.T. and Pissarides, C.A. (2003), 'Taxes, Subsidies and Equilibrium Labor Market Outcomes', in *Designing Inclusion: Tools to Raise Low-End Pay and Employment in Private Enterprise*, edited by Edmund Phelps, Cambridge University Press.

[40] Pavoni, N. (2009), 'Optimal Unemployment Insurance, with Human Capital Depreciation, and Duration Dependence', *International Economic Review*, 50(2), 323-362.

[41] Pissarides, C.A. (2000), *Equilibrium Unemployment Theory*, 2nd Edition, Cambridge, MA: MIT Press.

[42] Sattinger, M. (1995), 'General Equilibrium Effects of Unemployment Compensation with Labor Force Participation', *Journal of Labor Economics*, 13(4), 623-652.

[43] Shavell, S. and Weiss, L. (1979), 'The Optimal Payment of Unemployment Insurance Benefits over Time', *Journal of Political Economy*, 87(6), 1347-1362.

[44] Shimer, R. (2007), 'Reassessing the Ins and Outs of Unemployment', Working Paper, University of Chicago.

[45] Shimer, R. and Werning, I. (2006), 'On the Optimal Timing of Benefits with Heterogeneous Workers and Human Capital Depreciation', Working Paper, University of Chicago.

[46] Tirole, J. (2009), 'From Pigou to Extended Liability: On the Optimal Taxation of Externalities under Imperfect Financial Markets', *Review of Economic Studies*, Forthcoming.

[47] Topel, R.H. (1983), 'On Layoffs and Unemployment Insurance', *American Economic Review*, 73(4), 541-559.

[48] Topel, R. and Welch, F. (1980), 'Unemployment Insurance: Survey and Extensions', *Economica*, 47(187), 351-379.

[49] Wang, C. and Williamson, S.D. (2002), 'Moral Hazard, Optimal Unemployment Insurance and Experience Rating', *Journal of Monetary Economics*, 49, 1337-1371.

A Payroll Tax in First-Best Policy

Before deriving (25), it is necessary to rewrite the expression for the optimal value of b given by equation (9).

$$\begin{aligned}
b &= y - c\theta u - z(1-u) \\
&= y - c\theta u - \left[R - \frac{\eta(\theta)}{1-\eta(\theta)}c\theta + \frac{\lambda}{\rho+\lambda}\int_R^1 (s-R)dG(s)\right](1-u) \\
&= (1-u)\frac{\rho}{\rho+\lambda}\left[\frac{y}{1-u} - R\right] + \frac{\eta(\theta)}{1-\eta(\theta)}c\theta(1-u) + \lambda G(R)(1-u)\frac{1-R}{\rho+\lambda} - c\theta u \\
&= (1-u)\frac{\rho}{\rho+\lambda}\left[\frac{y}{1-u} - R\right] + \theta q(\theta)\left[\frac{1-R}{\rho+\lambda} - \frac{c}{q(\theta)}\right]
\end{aligned}$$

The second line was derived by using the optimal job destruction condition (11) to get rid of z. The obtain the third line, and to get rid of the integral, I have used the expression for the steady state level of output $y = (1-u)\left[G(R) + \int_R^1 sdG(s)\right]$ and then rearranged the terms. Finally, to get the last line, I have used equation (12) to rewrite the second term of the third line and used the fact that, in steady state, $\lambda G(R)(1-u) = \theta q(\theta)u$ to rewrite the third term of the third line.

Substituting this expression for b in (24) and using again the expression for the steady state level of unemployment, $\lambda G(R)(1-u) = \theta q(\theta)u$, yields equation (25).

B Wage Determination under Surplus Splitting

An entrepreneur expects a net present value V from the stream of income generated by a vacancy which will eventually become filled; while an unemployed expects $\tilde{U}$. The initial value of a match to a firm and to a worker are denoted by $J_0(1)$ and $\tilde{W}_0(1)$, respectively. The corresponding subsequent values, after an idiosyncratic shock has reduced the productivity of the match to x, are $J(x)$ and $\tilde{W}(x)$. Importantly, as it is assumed that the dollar amount of the match surplus is split in fixed proportions between the worker and the firm, the value functions of a worker, i.e. $\tilde{U}$, $\tilde{W}_0(1)$ and $\tilde{W}(x)$, give his expected future earnings and abstract from risk aversion.

The Bellman equation for the value of a vacancy is:

$$rV = -c + q(\theta)\left[J_0(1) + H - V\right],$$

which is the same as before, cf. equation (17). The corresponding equation for the value of unemployment is:

$$r\tilde{U} = z + b + \theta q(\theta)\left[\tilde{W}_0(1) - \tilde{U}\right].$$

The initial wage being denoted by $w_0(1)$, the initial value of match to a firm and to a worker are, respectively, given by:

$$\begin{aligned} rJ_0(1) &= 1 - (w_0(1) + \tau) + \lambda \int_R^1 J(s)dG(s) - \lambda G(R)F - \lambda J_0(1), \\ r\tilde{W}_0(1) &= w_0(1) + \lambda \int_R^1 \tilde{W}(s)dG(s) + \lambda G(R)\tilde{U} - \lambda \tilde{W}_0(1). \end{aligned}$$

Finally, the corresponding values for a subsequent match of productivity x, with wage $w(x)$, are:

$$\begin{aligned} rJ(x) &= x - (w(x) + \tau) + \lambda \int_R^1 J(s)dG(s) - \lambda G(R)F - \lambda J(x), \\ r\tilde{W}(x) &= w(x) + \lambda \int_R^1 \tilde{W}(s)dG(s) + \lambda G(R)\tilde{U} - \lambda \tilde{W}(x). \end{aligned}$$

The surplus splitting rule from which the initial wage, $w_0(1)$, is derived is:

$$(1-\beta)\left[\tilde{W}_0(1)-\tilde{U}\right]=\beta\left[J_0(1)+H-V\right],$$

where the firm receives the hiring subsidy in case an agreement is reached. The corresponding rule for an existing match that has just been hit by an idiosyncratic shock resulting in productivity x, and from which $w(x)$ is derived, is:

$$(1-\beta)\left[\tilde{W}(x)-\tilde{U}\right]=\beta\left[J(x)+F-V\right],$$

which takes into account the fact that, if the match dissolves, the firm needs to pay the layoff tax.

C Nash Bargaining with Commitment

Before solving the bargaining problem, it is useful to determine the value of the firm in match i at the job destruction threshold, $J_i(R_i)$. It could be deduced from the equation for $J(x)$, (13), that:

$$J_i(x)=\frac{x-R_i}{r+\lambda}+J_i(R_i).$$

Plugging this expression back into (13) evaluated at productivity R_i yields:

$$J_i(R_i)=\frac{1}{r+\lambda G(R_i)}\left[R_i-(w_i+\tau)+\frac{\lambda}{r+\lambda}\int_{R_i}^{1}(s-R_i)dG(s)-\lambda G(R_i)F\right].$$

Thus:

$$\frac{\partial J_i(1)}{\partial w_i}=-\frac{1}{r+\lambda G(R_i)},$$

and:

$$\frac{\partial J_i(1)}{\partial R_i}=-\frac{\lambda g(R_i)}{\left[r+\lambda G(R_i)\right]^2}\left[R_i-(w_i+\tau)+rF+\frac{\lambda}{r+\lambda}\int_{R_i}^{1}(s-R_i)dG(s)\right],$$

where $g(R)\equiv dG(R)/dR$.

Similarly, it could be deduced from the value function of the employed worker, $rW_i = v(w_i)+\lambda G(R_i)\left[U-W_i\right]$, that:

$$\frac{\partial W_i}{\partial w_i}=\frac{v'(w_i)}{r+\lambda G(R_i)},$$

and:

$$\frac{\partial W_i}{\partial R_i}=\frac{\lambda g(R_i)}{\left[r+\lambda G(R_i)\right]^2}\left[rU-v(w_i)\right].$$

Using the symmetry that prevails in equilibrium, i.e. $w = w_i$ and $R = R$, together with the value of unemployment, (54), this last expression simplifies to:

$$\frac{\partial W_i}{\partial R_i} = -\frac{\lambda g(R)}{[r + \lambda G(R)]} \frac{v(w) - v(z+b)}{r + \lambda G(R) + \theta q(\theta)}.$$

Finally, the first-order conditions for the wage w_i and the threshold R_i are obtained by differentiating the logarithm of the Nash product in (56). This yields:

$$\frac{\beta}{W_i - U} \frac{\partial W_i}{\partial w_i} = \frac{1-\beta}{J_i(1) + H - V} \left(-\frac{\partial J_i(1)}{\partial w_i} \right),$$

and:

$$\frac{\beta}{W_i - U} \frac{\partial W_i}{\partial R_i} = \frac{1-\beta}{J_i(1) + H - V} \left(-\frac{\partial J_i(1)}{\partial R_i} \right).$$

Using symmetry, i.e. dropping the subscript i, and substituting $V = 0$, $J(1)+H = c/q(\theta)$, $W - U = [v(w) - v(z+b)]/[\rho + \lambda G(R) + \theta q(\theta)]$ and the above derivatives into these first-order conditions yields (57) and (58).

D Search Intensity under Surplus Splitting

When wages are given by (67) and (68), the value of employment to workers satisfies:

$$\begin{aligned}
\rho W_0(1) &= v(w_0(1)) + \lambda \int_R^1 W(s) dG(s) + \lambda G(R) U - \lambda W_0(1), \\
\rho W(x) &= v(w(x)) + \lambda \int_R^1 W(s) dG(s) + \lambda G(R) U - \lambda W(x),
\end{aligned}$$

where the former expression corresponds to newly employed workers and the latter to those who have already been hit by an idiosyncratic shock. Subtracting the former from the latter, I obtain:

$$W(x) = W_0(1) - \frac{v(w_0(1)) - v(w(x))}{\rho + \lambda}.$$

Inserting this back into the expression for $W_0(1)$, yields:

$$\rho W_0(1) = \left[1 - \frac{\lambda}{\rho + \lambda} [1 - G(R)] \right] v(w_0(1)) + \frac{\lambda}{\rho + \lambda} \int_R^1 v(w(x)) dG(x) + \lambda G(R) [U - W_0(1)].$$

The value of unemployment for an average search intensity solves:

$$\rho U = v(z+b) - \sigma(s) + \theta q(\theta, s) [W_0(1) - U].$$

Taking the difference between these last two value equations gives:

$$W_0(1) - U = \frac{\left[1 - \frac{\lambda}{\rho+\lambda}\left[1 - G(R)\right]\right] v(w_0(1)) + \frac{\lambda}{\rho+\lambda}\int_R^1 v(w(x))dG(x) - v(z+b) + \sigma(s)}{\rho + \lambda G(R) + \theta q(\theta, s)}.$$

Finally, this should be substituted into the first-order condition for search intensity:

$$s\sigma'(s) = \theta q(\theta, s)\left[W_0(1) - U\right].$$

This yields (69) together with (70).